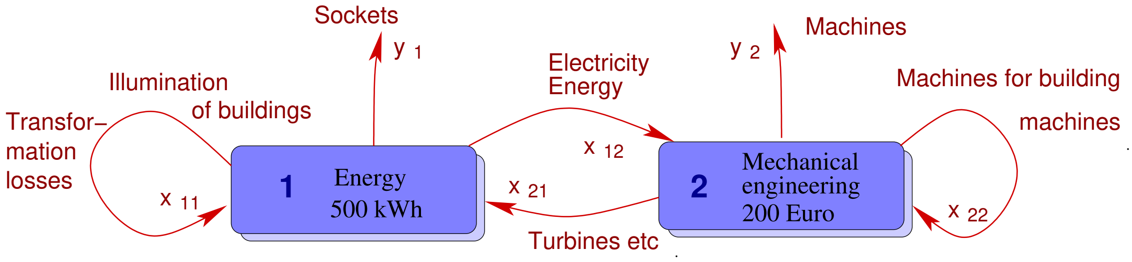
Methods in Transportation Econometrics and Statistics (Master)

Winter semester 2024/25, Solutions to Tutorial No. 12

Solution to Problem 12.1: Econometric Input-Output-Model (IOM)

(a) Diagram of flows:



The total outputs are given by the problem statement:

$$X_1 = 500\,\text{kWh} = 125\,€, \quad X_2 = 200\,€.$$

In order to ensure the steady stated in machine and electricity production, the inter-sectoral flows must satisfy (see problem statement)

$$\begin{aligned} X_{11} &= 0.1 * 500\,\text{kWh} = 50\,\text{kWh} = 12.5\,€, \\ X_{12} &= \frac{1\,\text{kWh}}{4\,€} * 200\,€ = 50\,\text{kWh} = 12.5\,€, \\ X_{21} &= \frac{1\,752\,€}{1\,\text{kW}} * 1\,\text{kW}/87\,600\,\text{kWh} * 500\,\text{kWh} = 10\,€, \\ X_{22} &= 0.05 * 200\,€ = 10\,€. \end{aligned}$$

(b) The external sectors and the end consumer get the difference between the total production and the intersectoral flows:

$$Y_i = X_i - \sum_j X_{ij}, \text{i.e.,}$$

- Energy:

  $Y_1 = X_1 - X_{11} - X_{12} = (500 - 50 - 50)\,\text{kWh} = 400\,\text{kWh} = 100\,€$

- Mech engineering:

  $Y_2 = X_2 - X_{21} - X_{22} = (200 - 10 - 10)\,€ = 180\,€$

(c) The IO-coefficients are obtained from the defining balance

$$X_i = \sum_{j=1}^{n} X_{ij} + Y_i = \sum_{j=1}^{n} A_{ij} X_j + Y_i$$

by

$$A_{ij} = \frac{X_{ij}}{X_j},$$

and, using the last results,

$$\underline{\underline{A}} = \begin{pmatrix} \frac{12.50}{125} & \frac{12.50}{200} \\ \frac{10}{125} & \frac{10}{200} \end{pmatrix} = \begin{pmatrix} 0.1000 & 0.0625 \\ 0.0800 & 0.0500 \end{pmatrix} \tag{1}$$

The coefficient matrix $\underline{\underline{B}}$ of the final demand is given by

$$\underline{\underline{B}} = (\underline{\underline{1}} - \underline{\underline{A}})^{-1}.$$

Step by step:

$$\underline{\underline{1}} - \underline{\underline{A}} = \begin{pmatrix} 1-0.1 & 0-0.0625 \\ 0-0.08 & 1-0.05 \end{pmatrix} = \begin{pmatrix} 0.9 & -0.0625 \\ -0.08 & 0.95 \end{pmatrix}. \tag{2}$$

and with the general 2×2 matrix inversion formula

$$\mathbf{M}^{-1} = \frac{1}{\det \mathbf{M}} \begin{pmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{pmatrix}$$

we have

$$\det(\mathbf{1} - \mathbf{A}) = 0.85$$

and finally

$$\underline{\underline{B}} = (\underline{\underline{1}} - \underline{\underline{A}})^{-1} = \begin{pmatrix} 1.118 & 0.0735 \\ 0.0941 & 1.059 \end{pmatrix}$$

(d) We now have a sudden demand change (in €) by

$$\Delta\vec{Y} = \begin{pmatrix} 0 \\ 0.11 * 180 \end{pmatrix} = \begin{pmatrix} 0 \\ 19.8 \end{pmatrix}.$$

Because of the linearity of the model, the relation defining $\underline{\underline{B}}$ is valid for the increments as well, hence

$$\Delta\vec{X} = \begin{pmatrix} 1.118 & 0.0735 \\ 0.0941 & 1.059 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 19.8 \end{pmatrix} = \begin{pmatrix} 1.46 \\ 20.96 \end{pmatrix}.$$

With 11 % or 19.80 € more production of machines for the external demand, the additional total production of machines is given by 20.96 €. Furthermore, to maintain the steady state, the total energy production must raise as well by a monetary equivalent of 1.46 €. In relative terms, this means

- a percentaged increase of total electricity production by $\Delta X_1/X_1 = 1.46/125 = 1.17\,\%$,
- a percentaged increase of machine production by $\Delta X_2/X_2 = 20.96/200 = 10.48\,\%$.

Although the external demand for energy does not increase, the energy *production* must increase to keep the steady state since some of the energy is needed to produce the additional machines for the external demand *and* the risen inter-sectoral demand.