Methods in Transportation Econometrics and Statistics (Master)

Winter semester 2024/25, Tutorial No. 7

Problem 7.1: Choice probabilities in trinomial Logit and i.i.d. Probit models

A trip to a certain destination takes $T_1 = 40\,\text{min}$ by foot, $T_2 = 15\,\text{min}$ by bike, and $T_3 = 15\,\text{min}$ when using public transport (PT). Additionally, using PT implies ad-hoc costs of C_3. The deterministic utilities are given by

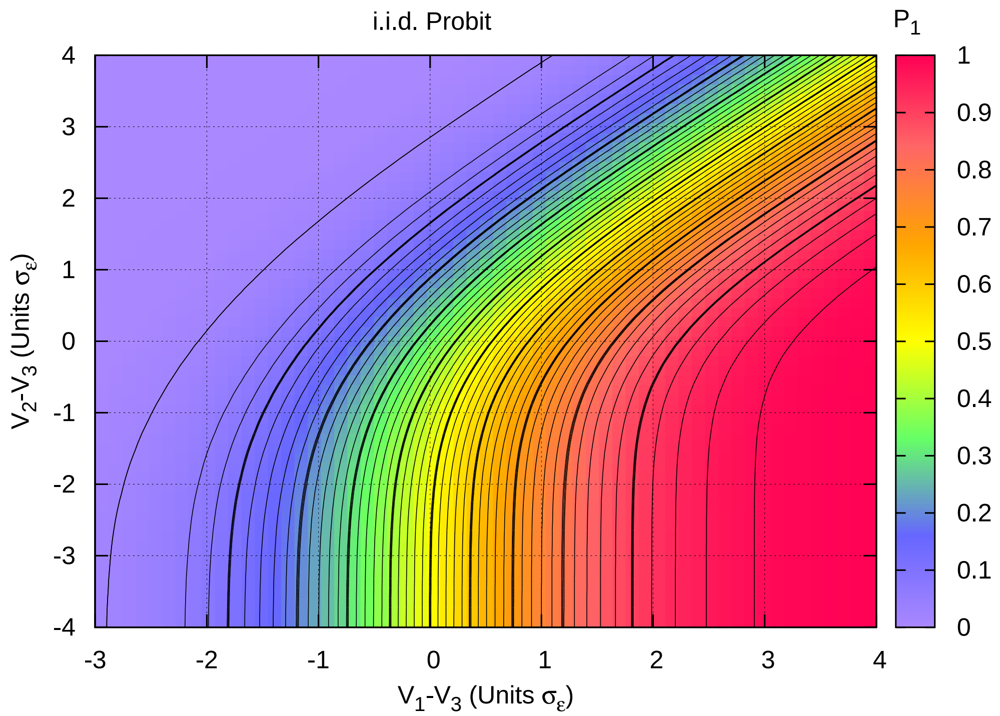
$$V_i = \beta_1 \delta_{i1} + \beta_2 \delta_{i2} + \beta_3 T_i + \beta_4 C_i$$

with following parameter values for the MNL: $\hat{\beta}_1 = -1, \hat{\beta}_2 = -2, \hat{\beta}_3 = -0.1, \hat{\beta}_4 = -1$.

(a) Which is the reference alternative for the ACs? Give the meaning of β_1 and β_2. Give the ansatz (generic or alternative-specific?) used to model the travel times.

(b) Give the utility unit (UU) in terms of time differences and ad-hoc costs and derive the implied value of time (VoT)

(c) Calculate the MNL choice probabilities analytically.



(d) Now use the i.i.d. multinomial-Probit model (MNP) assuming that all Logit parameters (and also the Logit utilities V_i) are multiplied by $\sqrt{6}/\pi$ reflecting the standard deviation of the standard-normal random utility $\sqrt{V(\epsilon)} = 1$ instead of $\pi/\sqrt{6}$ for the MNL. Read off the choice probability P_1 of the i.i.d. multinomial-Probit model (MNP) from the contour plot and the other probabilities by the relations $P_2(V_1 - V_3, V_2 - V_3) = P_1(V_2 - V_3, V_1 - V_3)$ und $P_3 = 1 - P_1 - P_2$.

(e) Discuss the small differences between the MNL and MNP probabilities.

(f) By some political initiative, PT is now freely available for all. Show that, in the MNL, some decision makers switch to the PT such that the relative attractivity P_1/P_2 remains constant (IIA property) while this is not the case for the MNP.

Problem 7.2: Revealed choice: survey in the audience

A survey among the students about the realized mode decision on the way to this lecture gives the following data:

Klasse \ Modus	$i=1$ (ped)	$i=2$ (bike)	$i=3$ (PT)	$i=4$ (car)
$n=1$: 0-2 km, no bike availability	2	–	3	1
$n=2$: 0-2 km, bike available	1	5	1	0
$n=3$: 2-6 km, no bike availability	1	–	2	1
$n=4$: 2-6 km, bike available	0	7	6	0
$n=5$: 6-10 km	0	2	8	3
$n=6$: 10-20 km	0	0	1	5

As only socioeconomic variable, the bike availability was recorded. Furthermore, the values of the only exogenous variable (distance) have been agggregated into classes.[1] The choice is modelled with the MNL specified as

$$V_{ni}(\vec{\beta}) = \beta_1 r_n \delta_{i1} + \beta_2 r_n \delta_{i2} + \beta_3 r_n \delta_{i3} + \beta_4 \delta_{i1} + \beta_5 \delta_{i2} + \beta_6 \delta_{i3} + V_{ni}^{\text{bike}}, \tag{1}$$

where r_i denotes the distance for person group n in kilometers, and the selector-dummy $\delta_{ij} = 1$ for $i = j$ and $= 0$, otherwise.

(a) Give the meaning of the parameters β_4 to β_6. Why would an additional factor $\beta_7 \delta_{i4}$ lead to a mis-specification?

(b) Give a parameter-free expression for V_{ni}^{bike} such that the bike mode can only be chosen if one is available.

(c) Show that Expressions (1) can be interpreted as a nonlinear function for the total travel time $T_4 - T_1$ where the total travel times T_i are of the form

$$V_{ni} = -T_{ni} = -\left(T_i^{(0)} + \frac{r_n}{v_i} \right).$$

Identify the parameters β_1 to β_6 with functions of the setup times $T_i^{(0)}$ and speeds v_i.

Hint: Notice that $V_4 = 0$, i.e., the reference alternative. Therefore, you need to formulate the utility differences $V_1 - V_4$ in terms of the travel times.

[1] In a real investigation, every person is asked individually, so this would not be necessary.