Immatriculation nummer: (please also write it on all your work sheets)

Exam to the Lecture Traffic Dynamics and Simulation SS 2026

Total 120 points

Problem 1 (30 points)

Given is a freeway with two lanes in the considered direction and a narrow road-work section where a maximum speed of 60 km/h is prescribed and, additionally, the average time gap of the drivers increases from 1.2 s to 1.4 s. The average effective length of all vehicles is determined to be $l_{\text{eff}} = 1/\rho_{\text{max}} = 8\,\text{m}$. Outside of the road-work section, the average free-flow speed is observed to be 120 km/h.

(a) Determine the capacity of the road outside and inside of the roadworks section (both lanes together) assuming a triangular fundamental diagram and argue that an inflow of 3 600 veh/h will not lead to a breakdown.

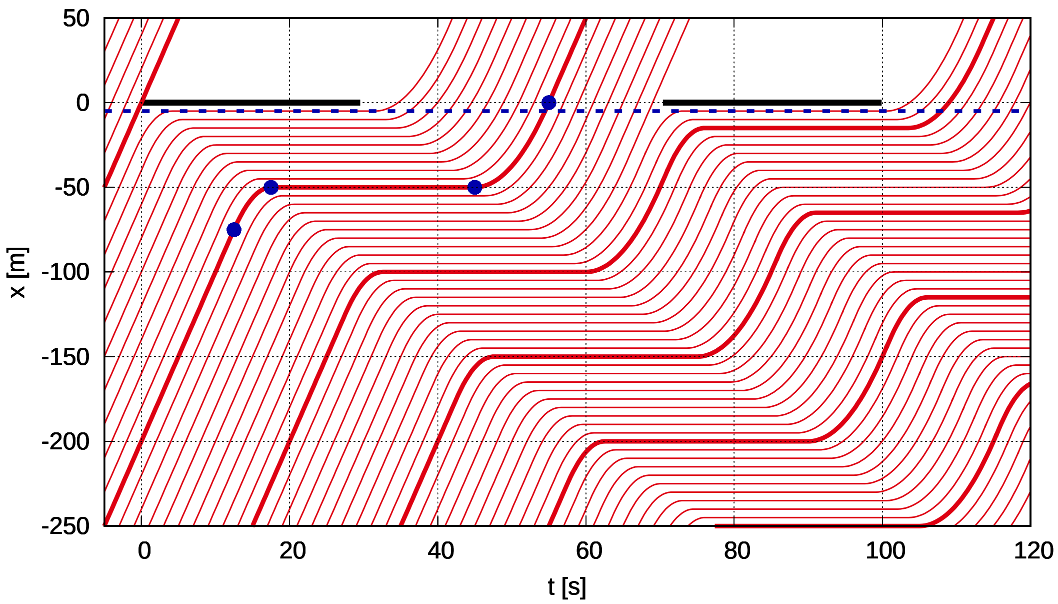
(b) At $t = 0$, a surge of the demand from 3 600 veh/h to 4 500 veh/h occurs. Show that the roadworks section cannot accommodate the demand and a breakdown to congested traffic ensues.

(c) Determine flow, density, and speed inside the congested zone.

(d) Calculate the propagation velocity of the upstream jam front where drivers enter the congested zone.

Problem 2 (40 points)

Given are trajectories of cars near a signalized intersection with the traffic light at $x = 0$ (red for the times where the thick black lines appear). Every 10^{th} trajectory is drawn as a thicker line.



(a) Determine the maximum density ρ_{max} during the queuing phases and the maximum outflow $Q_{\text{max}} = Q_{\text{out}}$ at $x = 0\,\text{m}$ during the green phases.

(b) Determine, for $x < -100\,\text{m}$ and $t < 20\,\text{s}$, the flow, density, and speed.

(c) Determine the cycle-average capacity of the signalied intersection. Is this capacity sufficient for the actual demand?

(d) Determine the wave velocity w of the congested $\to$ free transitions.

(e) Calculate the total starting delay of the first vehicles when the traffic light turns green by comparing the theoretical maximum number of vehicles passing during one green phase (outflow times green phase) with the actual number.

(f) The stationary detector at $x = -5\,\text{m}$ measures an arithmetic speed average over one signal cycle of $8\,\text{m/s}$. Give estimates for flow and density from this detector when aggregating over one cycle

(g) Calculate Edie's definitions for flow, density, and speed for a spatiotemporal area $A = [-7.5\,\text{m}, -2.5\,\text{m}] \times [0\,\text{s}, 70\,\text{s}]$. Assume a total travel time $T_{\text{tot}} = 48\,\text{s}$ in A.

(h) Determine the (constant) deceleration and acceleration in the approaching and starting phases, respectively.

Hint: For one trajectory, bullets indicate the begin and end of the braking and acceleration phases.

Problem 3 (50 points)

The acceleration equation of the IDM+ is as follows

$$\frac{\mathrm{d}v}{\mathrm{d}t} = f(s, v, v_l) = a \min\left[1 - \left(\frac{v}{v_0}\right)^4, \; 1 - \left(\frac{s^*(v, v_l)}{s}\right)^2\right]$$

with the dynamic desired gap

$$s^*(v, v_l) = s_0 + vT + \frac{v(v - v_l)}{2\sqrt{ab}}.$$

(a) Describe the workings of this model: its inputs, its output and its model parameters.

(b) Derive the steady-state speed-density relation

(c) Argue that no vehicle length is needed for a stability analysis

(d) What is the difference between local and string stability?

(e) The conditions for local and string stability depend on the sensitivities f_s, f_v, and f_l of the acceleration function f with respect to the gap s, speed v, and leading speed v_l, respectively, as follows.

$$\begin{aligned} \text{local stability:} \quad & f_v \leq 0, \\ \text{string stability:} \quad & v_e' \leq \frac{1}{2}\left(f_l - f_v\right). \end{aligned}$$

Show that, for free traffic ($s_e > s_0 + v_0 T$), both conditions are always satisfied.

(f) For interacting steady-state traffic, $s_e < s_0 + v_0 T$ or $v_e < v_0$, the gradient of the speed-gap relation $v_e(s) = (s - s_0)/T$ and the acceleration sensitivities are given as follows:

$$v_e'(s) = \frac{1}{T}, \quad f_s = \frac{2a}{s}, \quad f_v = -\sqrt{\frac{a}{b}}\frac{v_e(s)}{s} - \frac{2aT}{s}, \quad f_l = +\sqrt{\frac{a}{b}}\frac{v_e(s)}{s}.$$

Derive the local and string stability conditions for nearly stopped traffic, $s_e/s_0 - 1 \ll 1$.

(g) Assume now the IDM+ parameter values $v_0 = 30\,\mathrm{m/s}$, $T = 1.5\,\mathrm{s}$, $s_0 = 2\,\mathrm{m}$, and $a = b = 2\,\mathrm{m/s^2}$. Determine the steady-state gap regions where the IDM+ is string stable and string unstable.