# Traffic Flow Dynamics and Simulation

## Summer semester, Solutions to Work Sheet 10, page 1

### Solution to Problem 10.1: Why the grass is always greener on the other side

We assume two lanes with staggered regions of highly congested traffic $(\rho_1, V_1)$ and less congested traffic ($\rho_2 < \rho_1$, $V_2 > V_1$) of the same length: Whenever there is highly congested traffic on lane 1, congestion is less on lane 2, and vice versa (cf. the figure in the problem statement). Since traffic in both regions is (more or less) congested and the fundamental diagram is triangular by assumption, the transitions from region 1 and 2 and from 2 to 1 remain sharp and propagate according to the shock-wave formula (**??**) at a constant velocity

$$c = \frac{Q_2 - Q_1}{\rho_2 - \rho_1} = -\frac{l}{T} = -5\,\mathrm{m/s}.$$

The fraction of time in which drivers are stuck in the highly congested regions is obviously equal to the fraction of time spent in regions of type 1. Denoting by $\tau_i$ the time intervals $\tau_i$ to pass one region $i = 1$ or 2, we express this fraction by

$$p_{\text{slower}} = p_1 = \frac{\tau_1}{\tau_1 + \tau_2}\,.$$

When evaluating $\tau_i$, it is crucial to realize that the regions propagate in the opposite direction to the vehicles, so the relative velocity $V_i + |c|$ is relevant. Assuming equal lengths $L$ for both regions, the passage times are $\tau_i = L/(V_i + |c|)$, so

$$p_1 = \frac{\frac{L}{V_1+|c|}}{\frac{L}{V_1+|c|} + \frac{L}{V_2+|c|}} = \frac{V_2 + |c|}{V_2 + V_1 + 2|c|}.$$

For example, if $V_1 = 0$ and $V_2 = 10\,\mathrm{m/s}$, the fraction is

$$p_1 = \frac{10 + 5}{10 + 10} = \frac{3}{4},$$

i.e., drivers are stuck in the slower lane 75 % of the time – regardless which lane they choose or of whether they change lanes or not.

Alternatively, one picks out a vehicle at random. Since the less and highly congested regions have the same length, the fraction of vehicles in the highly congested region, i.e., the probability of picking one from this region, is given by

$$p_1 = \frac{\rho_1}{\rho_1 + \rho_2} = \frac{200}{200 + 200/3} = \frac{3}{4}.$$

## Solution to Problem 10.2: Stop or cruise? 1. yellow (amber) time intervals

We distinguish two cases: (i) Drivers can pass the traffic light at unchanged speed in the yellow phase, i.e.,

$$s < s_1 = v\tau_y.$$

(ii) When cruising, drivers would pass the traffic light in the red phase, so stopping is mandatory. In this case, drivers need a reaction time $T_r$ to perceive the signal, make a decision, and stepping on the braking pedal. Afterwards, we assume that they brake at a constant deceleration $b$ so as to stop just at the stopping line. This results in the *stopping distance*

$$s = vT_r + \frac{v^2}{2b}. \tag{1}$$

Obviously, the *worst case* for the initial distance $s$ to the stopping line at switching time green-yellow is the threshold $s = s_1$ between (i) and (ii), i.e., cruising is just no more legal. Inserting $s = s_1$ into Eq. (1) and solving for $b$ gives

$$b = \frac{v}{2(\tau_y - T_r)} \quad = \quad 3.47\,\mathrm{m/s^2}.$$

This is a significant, though not critical, deceleration. It is slightly below the deceleration $3.86\,\mathrm{m/s^2}$ implied by the braking distance rule „speedometer reading in km/h squared divided by 100“ (cf. Tutorial 8) but above typical comfortable decelerations of the order of $2\,\mathrm{m/s^2}$. We conclude that the legal minimum duration of yellow phases is consistent with the driver and vehicle capabilities.

## Solution to Problem 10.3: Stop or cruise? 2. decisions implied by car-following models

(a) This MOBIL decision criterion means that one brakes to a stop whenever the braking deceleration at decision time is smaller than the safe braking deceleration. Otherwise one cruises through the intersection. In sensible models as the IDM, one tries to bring the situation under control whenever the required braking deceleration is above the comfortable deceleration $b$. Assuming $b_{\text{safe}} > b$ this means that the initial deceleration is the highest, so a safe deceleration to a stop is guaranteed.

(b) If the decision is “stop”, then one assumes a virtual standing vehicle of length zero at the stopping line, i.e., $v_l = 0$ or $\Delta v = v$. Evaluating the decision criterion for the IDM gives

$$\begin{aligned}
\frac{\mathrm{d}v}{\mathrm{d}t} = -a\left(\frac{s^*(v,v_l)}{s}\right)^2 &\quad > \quad -b_{\text{safe}} \\
a\left(\frac{s^*(v,v_l)}{s}\right)^2 &\quad < \quad b_{\text{safe}} \\
a\frac{(s^*(v,v_l))^2}{b_{\text{safe}}} &\quad < \quad s^2
\end{aligned}$$

This means, the general critical gap is given by

$$s > s_{\text{safe}}(v) = s^*(v,0)\sqrt{\frac{a}{b_{\text{safe}}}}. \tag{2}$$

(c) For $a = b = b_{\text{safe}}$, we obtain for $v = v_0$

$$s > s_{\text{safe}}(v) = s^*(v,0) = s_0 + v_0 T + \frac{v_0^2}{2b},$$

If the time gap parameter $T$ also gives the reaction time, this is precisely the minimum gap $s_0$ plus the stopping distance with its components reaction distance $vT$ and braking distance $v^2/(2b)$!

Specific values:

- $v_0 = 50\,\text{km/h}: \; s_{\text{safe}} = s^*(v_0,0) = 62\,\text{m},\; \Delta t_{\text{safe}} = s_{\text{safe}}/v_0 = 4.47\,\text{s}$
- $v_0 = 70\,\text{km/h}: \; s_{\text{safe}} = s^*(v_0,0) = 114\,\text{m},\; \Delta t_{\text{safe}} = s_{\text{safe}}/v_0 = 5.86\,\text{s}$

(d) The above critical distances and associated critical time intervals till passing are too great. Considering that the minimum amber times for 50 km/h and 70 km/h are given by 3 s and 4 s, respectively, this strategy may lead to crossing red traffic lights. Of course, the reason is that the legislation imposes on the driver a safe deceleration $b_{\text{safe}}$ that is somewhat greater than the comfortable deceleration $b$. In this case, we obtain from the above general formula (2) for $b_{\text{safe}} = 4\,\text{m/s}^2$,

- $v_0 = 50\,\text{km/h}: \; s_{\text{safe}} = 44\,\text{m},\; \Delta t_{\text{safe}} = s_{\text{safe}}/v_0 = 3.16\,\text{s}$
- $v_0 = 70\,\text{km/h}: \; s_{\text{safe}} = s^*(v_0,0) = 81\,\text{m},\; \Delta t_{\text{safe}} = s_{\text{safe}}/v_0 = 4.14\,\text{s}$

There is still a minimal chance of passing a red traffic light if the yellow/amber times are at their minimum allowed values of 3 s and 4 s, respectively. This is due to an IDM imperfection: At the beginning of a stopping maneuver, the IDM tends to “brake” a little too hard

(e) For the OVM, we have the critical gap

$$\dot{v}_{\text{OVM}} = \frac{v_{\text{opt}}(s) - v}{\tau} < -b_{\text{safe}}$$

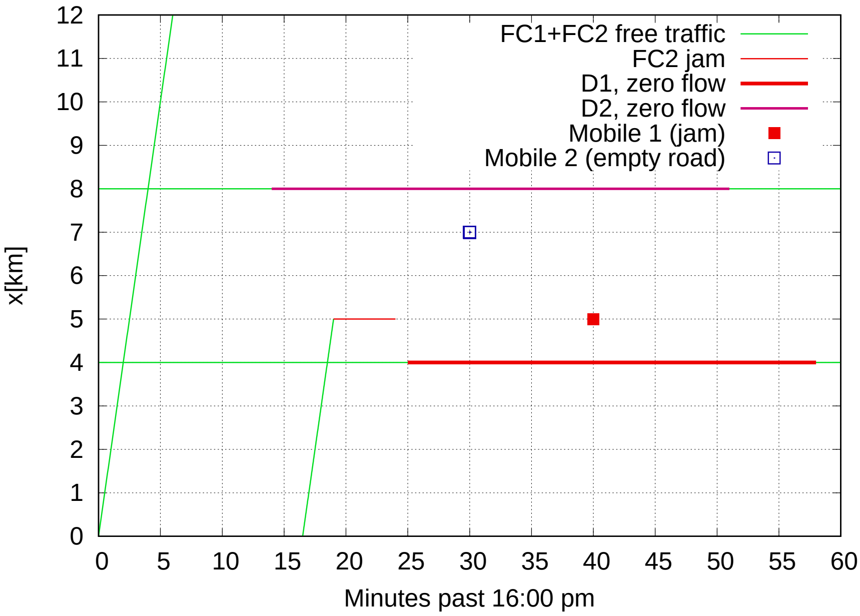
Because only the interacting range $s < v_0 T$ is relevant, this leads to

$$\dot{v}_{\text{OVM}} = \frac{s/T - v}{\tau} > -b_{\text{safe}}, \quad \Rightarrow \quad s > s_{\text{safe}} = T\,(v - \tau b_{\text{safe}})$$

or $s > s_{\text{safe}} = T(v - \tau b_{\text{safe}})$ For $v = v_0 = 72\,\text{km/h} = 20\,\text{m/s}$, $\tau = T/2 = 0.5\,\text{s}$, and $b_{\text{safe}} = 4\,\text{m/s}^2$, this results in $s_{\text{safe}} = 9.0\,\text{m}$: This is much too low: For example, one would brake if $s = 10\,\text{m}$. However, at this gap, the kinematic braking deceleration to avoid crossing the stopping line (and stopping mid-intersection instead) is given by $b_{\text{kin}} = v^2/(2s) = 20\,\text{m/s}^2$.

## Solution to Problem 10.4: Reconstruction of the traffic situation around an accident

(a) In the space-time diagram below, thin dashed green lines mark confirmed free traffic while all other information is visualized using thicker lines and different colors. The respective information is denoted in the key. The signal „zero flow" means „I do not know; either empty road or stopped traffic".




(b) The information of the first floating car (FC1) tells us the speed in free traffic, $V_{\text{free}} = 10\,\text{km}/5\,\text{minutes} = 120\,\text{km/h}$. From the second floating car (FC2) we know that an upstream jam front passes $x = 5\,\text{km}$ at 4:19 pm.

The stationary detectors D1 at $x = 4\,\text{km}$ and D2 at 8 km both report zero flows in a certain time interval but this does not tell apart whether the road is maximally congested or empty. However, we additionally know by the two mobile phone calls that the road is fully congested at 5 km while it is empty at 7 km. The congestion at 5 km is also consistent with the trajectory of the second floating car. Since downstream jam fronts (transition jam $\to$ free traffic) are either stationary or propagate upstream at velocity $c \approx -15\,\text{km/h}$ but never downstream (apart from the special case of a moving bottleneck), we know that the missing vehicle counts of D1 are the consequence of standing traffic while that of D2 reflect an empty road (at least when ignoring the possibility that there might be another obstruction more downstream causing a second jam).

With this information, we can estimate the motion of the upstream jam front. Assuming a constant propagation velocity $c^{\text{up}}$, we determine this velocity from the spatiotemporal points where detector D1 and the second floating car encounter congestion, respectively:

$$c^{\text{up}} = \frac{-1\,\text{km}}{6\,\text{min}} = -10\,\text{km/h}\,.$$

The motion of this front is another strong evidence that D2 does not measure a transition from free to fully congested traffic but from free traffic to no traffic at all at $x = 8\,\text{km}$ and $t =$ 4:14 pm: Otherwise, the propagation velocity $c^{\text{up}}$ would be $-4\,\text{km}/5\,\text{min} = -48\,\text{km/h}$ in this region which is not possible even if we do not require $c^{\text{up}}$ to be constant: The largest possible negative velocity $c^{\text{up}}$, realized under conditions of maximum inflow against a full road block, is only insignificantly larger in magnitude than $|c^{\text{down}}| \approx 15\,\text{km/h}$.

Now we have enough information to determine location and time of the initial road block (accident). Intersecting the line

$$x^{\text{up}}(t) = 4\,\text{km} + c^{\text{up}} t - 25\,\text{min}) = 4\,\text{km} - \frac{t - 25\,\text{min}}{6}\,\text{km/min}$$

characterizing the upstream front with the trajectory $x_{\text{last}}(t)$ of the last vehicle that made it past the accident location,

$$x_{\text{last}}(t) = 8\,\text{km} + v_0(t - 14\,\text{min}) = 8\,\text{km} + (t - 14\,\text{min})\,2\,\text{km/min}$$

yields location and time of the road block:

$$x_{\text{crash}} = 6\,\text{km}, \quad t_{\text{crash}} = \text{4:13 pm}\,.$$

(c) After the accident site is cleared, the initially stationary downstream jam front (fixed at the accident site) starts moving at the characteristic velocity $c^{\text{down}} = -15\,\text{km/h} = -1\,\text{km}/4\,\text{min}$. Since the detector at $x = 4\,\text{km}$ (D1) detects non-zero traffic flow from 4:58 pm onwards, the front is described by

$$x^{\text{down}}(t) = 4\,\text{km} + c^{\text{down}}(t - 58\,\text{min}).$$

Obviously, the accident location ($x_{\text{crash}} = 6\,\text{km}$) is cleared exactly at the time where the moving downstream jam front crosses the accident site (cf. the figure), i.e., at $t_{\text{clear}} =$4:50 pm.