Traffic Flow Dynamics and Simulation

Summer semester, Solutions to Work Sheet 9, page 1

Solution to Problem 9.1: Dynamics of a single vehicle between two signalized intersections

(a) *parameters.* The free acceleration is the same as that of the OVM. Hence, v_0 is the desired speed and τ the adaptation time. If the model decelerates, it does so with the deceleration b. Since, at this deceleration, the kinematic braking distance to a complete stop is given by $\Delta x_{\text{brake}} = v^2/(2b)$, the vehicle stops at a distance s_0 to the (stopping line of) the red traffic light. This explains the meaning of the last parameter. Notice that, in this model, vehicles would follow any leading vehicle driving at a constant speed $v_l < v_0$ at the same gap s_0, i.e., the model does not include a safe gap. Nor does it contain a reaction time. The model is accident-free with respect to stationary obstacles, but not when slower vehicles are involved.

(b) *free acceleration phase.* Here, the first condition of the model applies, so we have to solve the ordinary differential equation (ODE) for the speed

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{v_0 - v}{\tau} \quad \text{with } v(0) = 0.$$

The exponential ansatz $\mathrm{e}^{\gamma t}$ for the homogeneous part $\frac{\mathrm{d}v}{\mathrm{d}t} = -v/\tau$ gives the solvability condition $\gamma = 1/\tau$. Furthermore, the general solution for the full inhomogeneous (ODE) reads

$$v(t) = A\mathrm{e}^{-t/\tau} + B.$$

The asymptotic $v(\infty) = B = v_0$ yields the inhomogeneous part B. Determining the integration constant A by the initial condition $v(0) = A + B = A + v_0 = 0$ gives $A = -v_0$, so the speed profile reads

$$v(t) = v_0\left(1 - \mathrm{e}^{-\frac{t}{\tau}}\right).$$

Once $v(t)$ is known, we determine the trajectory $x(t)$ by integrating over time. With $x(0) = 0$, we obtain

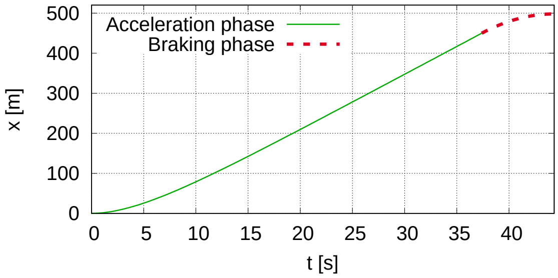
$$\begin{aligned} x(t) &= \int_0^t v(t')\ \mathrm{d}t' = v_0 \int_0^t \left(1 - \mathrm{e}^{-\frac{t'}{\tau}}\right)\ \mathrm{d}t' \\ &= v_0\left[t' + \tau\mathrm{e}^{-\frac{t'}{\tau}}\right]_{t'=0}^{t'=t} = v_0 t + v_0\tau\left(\mathrm{e}^{-\frac{t}{\tau}} - 1\right). \end{aligned}$$

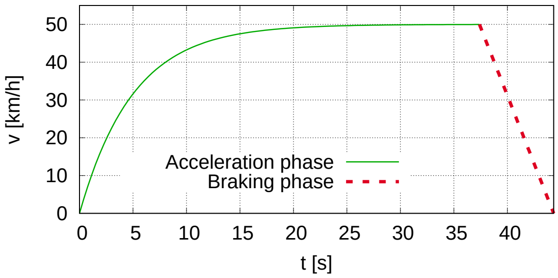
By identifying parts of this expression with $v(t)$, this simplifies to

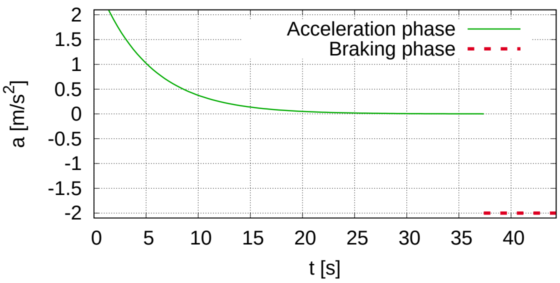
$$x(t) = v_0 t - v(t)\tau.$$

Finally, to obtain the acceleration profile, we either differentiate $v(t)$, or insert $v(t)$ into the right-hand side of the ODE. In either case, the result is

$$\dot{v} = \frac{v_0 - v}{\tau} = \frac{v_0}{\tau}\, \mathrm{e}^{-1/\tau}.$$







(c) *braking phase.* The red traffic light represents a standing virtual vehicle of zero length at the stopping line, so $\Delta v = v$. This phase starts at a distance

$$s_c = s_0 + \frac{v^2}{2b} = 50.2\,\mathrm{m}$$

to the stopping line, and the vehicle stops at a distance s_0 to this line.

(d) *trajectory.* For the accelerating phase, the trajectory has already been calculated. The deceleration phase begins at the location

$$x_c = L - s_c = L - s_0 - \frac{v^2}{2b} \approx 450\,\mathrm{m}.$$

To approximatively determine the time t_c at which the deceleration phase begins, we set $v(t_c) = v_0$ to obtain

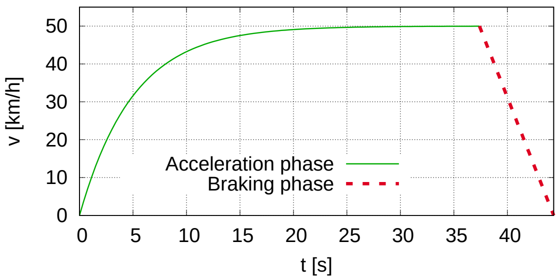
$$x_c(t_c) = v_0 t_c - v(t_c)\tau \approx v_0(t_c - \tau) \quad \Rightarrow \quad t_c = \frac{x_c}{v_0} + \tau = 32.4\,\mathrm{s} + 5.0\,\mathrm{s} = 37.4\,\mathrm{s}.$$

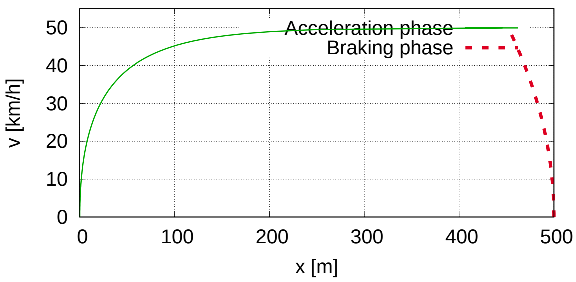
With the braking time v_0/b, this also gives the stopping time

$$t_{\text{stop}} = t_c + \frac{v_0}{b} = 44.3\,\mathrm{s}.$$

In summary, the speed profile $v(t)$ can be expressed by (cf. the graphics below)

$$v(t) = \begin{cases} v_0\left(1 - \mathrm{e}^{-t/\tau}\right) & 0 \le t < t_c, \\ v_0 - b(t - t_c) & t_c \le t \le t_{\text{stop}}, \\ 0 & \text{otherwise.} \end{cases}$$





Solution to Problem 9.2: Full Velocity Difference Model

(a) General plausibility arguments require the steady-state speed $v_{\text{opt}}(s)$ to approach the desired speed v_0 when the gap s tends to infinity. However, for an arbitrarily large distance to the red traffic light modeled by a standing virtual vehicle ($\Delta v = v$), the FVDM vehicle accelerates according to

$$\dot{v} = \frac{v_0 - v}{\tau} - \gamma v = \frac{v_0}{\tau} - \left(\frac{1}{\tau} + \gamma\right) v.$$

(i) From this it follows that the acceleration $\dot{v}$ becomes zero for a terminal speed

$$v^* = \frac{v_0}{1 + \gamma\tau}.$$

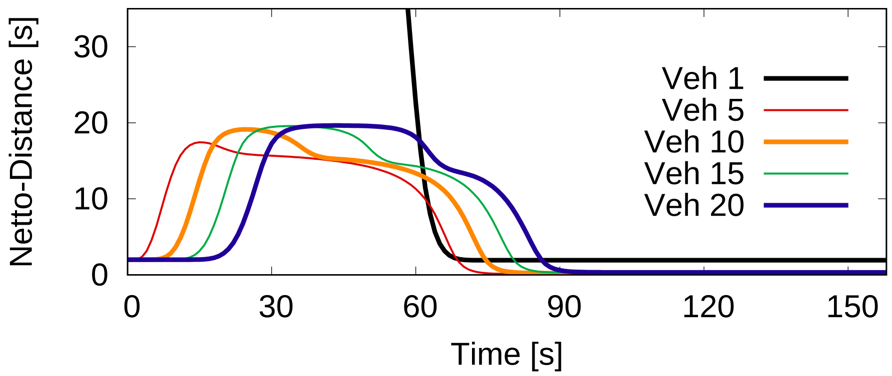
This is the maximum speed an initially standing FVDM vehicle can reach in this situation. It is significantly lower than v_0.

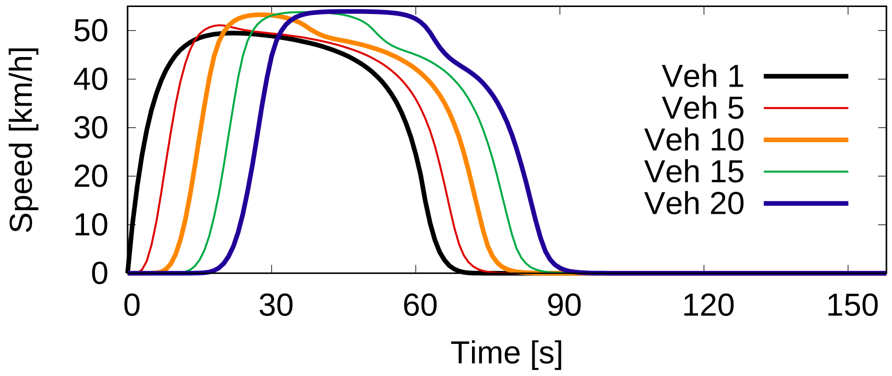
(ii) For the parameter values of the problem statement, $v^* = 13.5\,\mathrm{km/h}$ agreeing with the simulated speed time series.

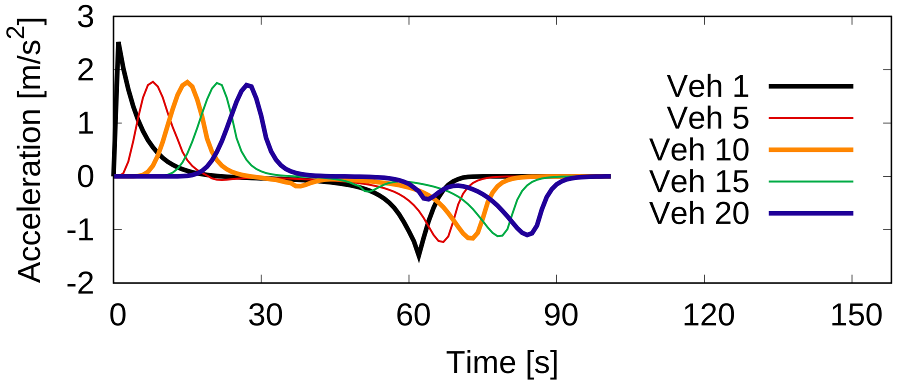
(b) The “improved FVDM”

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{v_{\mathrm{opt}}(s) - v}{\tau} - \gamma \frac{v - v_l}{\max[1, s/(v_0 T)]}$$

reverts to the normal OVM for gaps $s \to \infty$. Since the OVM does not show this inconsistency, the problem is solved while retaining the favourable property of the FVDM to include the relative speed $v - v_l$ resembling much more a realistic and safer driving style. For reference, the situation of the problem statement is shown for the modified FVDM:


Netto-Distance [s]
Time [s]
Veh 1
Veh 5
Veh 10
Veh 15
Veh 20


Speed [km/h]
Time [s]
Veh 1
Veh 5
Veh 10
Veh 15
Veh 20


Acceleration [m/s^2]
Time [s]
Veh 1
Veh 5
Veh 10
Veh 15
Veh 20

Solution to Problem 9.3: Reaction to vehicles merging into the lane

(a) *Reaction for the IDM:* For $v = v_0/2$, the IDM steady-state space gap reads

$$s_e(v) = \frac{s_0 + vT}{\sqrt{1 - \left(\frac{v}{v_0}\right)^\delta}} = \frac{s_0 + \frac{v_0 T}{2}}{\sqrt{1 - \left(\frac{1}{2}\right)^\delta}}.$$

The prevailing contribution comes from the prescribed time headway (for $s_0 = 2\,\mathrm{m}$ and $\delta = 4$, the other contributions only make up about 10 %). This problem assumes that the merging vehicle reduces the gap to the considered follower to half the steady-state gap, $s = s_e/2 = v_0 T/4$, while the speed difference remain zero. The new IDM acceleration of the follower (with $a = 1\,\mathrm{m/s^2}$ and $\delta = 4$) is therefore

$$\begin{aligned}
\dot{v}_{\mathrm{IDM}} &= a\left[1 - \left(\frac{v}{v_0}\right)^\delta - \left(\frac{s_0 + vT}{s}\right)^2\right] \\
&\overset{(v=v_0/2,\, s=s_e/2)}{=} a\left[1 - \left(\frac{1}{2}\right)^\delta - \left(\frac{s_0 + v_0T/2}{s_e/2}\right)^2\right] \\
&\overset{s_e(v)=s_e(v_0/2)}{=} -3a\left[1 - \left(\frac{1}{2}\right)^\delta\right] = -\frac{45}{16}\ \mathrm{m/s^2} = -2.81\,\mathrm{m/s^2}.
\end{aligned}$$

(b) *Reaction for the simplified Gipps' model:* For this model, the steady-state gap in the car-following regime reads $s_e(v) = v\Delta t$. Again, at the time of merging, the merging vehicle has the same speed $v_l = v = v_0/2$ as the follower, and the gap is half the steady-state gap, $s = (v\Delta t)/2 = v_0\Delta t/4$. The new speed of the follower is restricted by the safe speed v_{safe}:

$$v(t + \Delta t) = v_{\mathrm{safe}} = -b\Delta t + \sqrt{b^2(\Delta t)^2 + v_l^2 + 2b(s - s_0)} = 19.07\,\mathrm{m/s}.$$

This results in an effective acceleration

$$\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)_{\mathrm{Gipps}} = \frac{v(t + \Delta t) - v(t)}{\Delta t} \approx -0.93\,\mathrm{m/s^2}.$$

We conclude that the (simplified) Gipps' model describes a more relaxed driver reaction compared to the IDM. Notice that both the IDM and Gipps' model would generate significantly higher decelerations for the case of slower leading vehicles (dangerous situation).

Solution to Problem 9.4: Reactions to a traffic light turning red

At the beginning, we have the dynamic state variables

$$s = 50\,\mathrm{m}, \quad v = 15\,\mathrm{m/s}, \quad \Delta v = v - v_l 15\,\mathrm{m/s}$$

(a) *Reaction for the IDM:* The dynamic desired gap is given by

$$s^* = s_0 + vT + \frac{v^2}{2\sqrt{ab}} \approx 96.5\,\text{m}.$$

The IDM acceleration term is zero since the driver cruises at his/her desired speed. The braking term and thus the full IDM deceleration calculates to

$$\dot{v}_{\text{IDM}} = -a \left(\frac{s^*}{s}\right)^2 = \underline{\underline{-3.73 \text{ m/s}^2.}}$$

(b) *Reaction for the simplified Gipps model:* We have

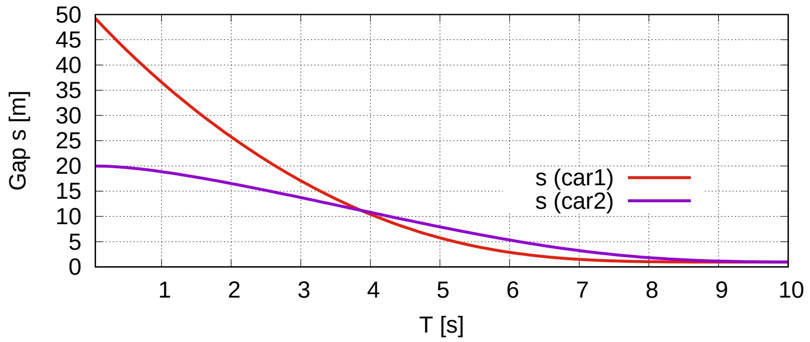
$$v_{\text{safe}} = -b\Delta t + \sqrt{b^2(\Delta t)^2 + 2b(s - s_0)} = 12.3\text{m/s}.$$

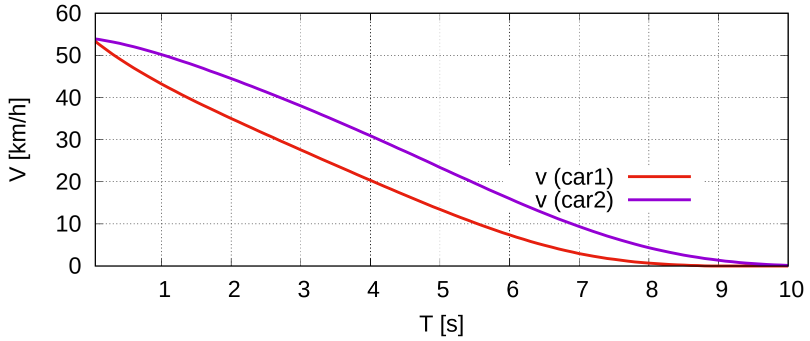
So, initially,

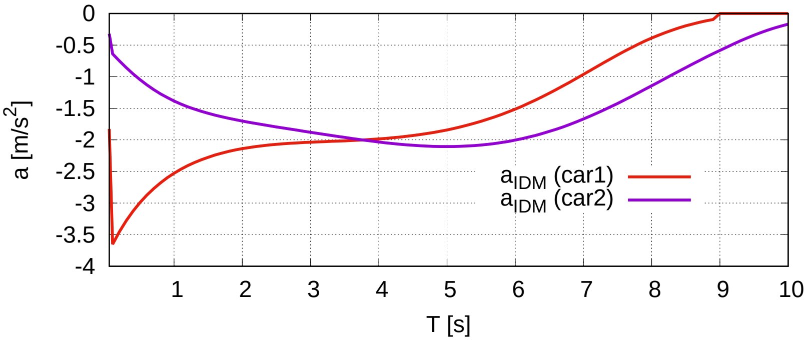
$$\dot{v}_{\text{Gipps}} = \frac{v_{\text{safe}} - v}{\Delta t} = \underline{\underline{-2.72 \text{ m/s}^2.}}$$

and $\dot{v} = -b$ after the first Δt-interval ensuring (as in the IDM) a safe stopping at a distance s_0 before the stopping line.

Following plots show the IDM controlled braking maneuvers of the subject vehicle and a further follower initially following at a gap of 20 m.


Gap s [m]
T [s]
s (car1)
s (car2)


V [km/h]
T [s]
v (car1)
v (car2)


a [m/s2]
T [s]
aIDM (car1)
aIDM (car2)