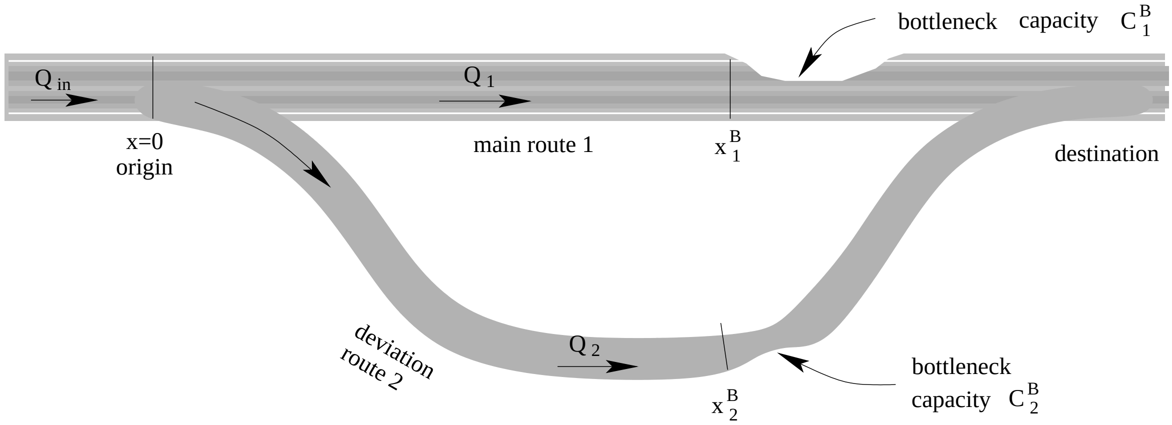
Traffic Flow Dynamics and Simulation

Summer semester, Tutorial 12, page 1

Problem 12.1: Dynamic Navigation

Given is a main route R1 (length from diverge to merge 15 km, 3 lanes, free speed $V_0 = 72\,\text{km/h}$, LWR wave velocity $w = -18\,\text{km/h}$, maximum density $\rho_{\text{max}} = 150\,\text{veh/km}$ per lane) and a deviation route R2 of length 16 km from diverge to merge, one lane, and the same values for V_0, w, and ρ_{max}:



Furthermore, R1 and R2 have bottlenecks of capacities $C_1^{\text{B}} = 4\,860\,\text{veh/h}$ and $C_2^{\text{B}} = 1\,080\,\text{veh/h}$, respectively that are located 14 km downstream of the merge (the precise position does not play a role, here). For simplicity, we assume no speed restrictions inside the bottlenecks. Use the LWR with a triangular funda,mental diagram for the following tasks.

(a) Determine the travel times T_{10} and T_{20} for the empty network.

(b) Determine the user equilibrium (UE) $T_1(x) = T_{20}$ by calculating the length x of the jam on R1.

(c) Argue that the system optimum (SO), $Q_1^{\text{tot}} T_1 + Q_2^{\text{tot}} T_2 \stackrel{!}{=} \min$ leads to the same flow $Q_1^{\text{tot}} = C_1^{\text{B}}$ on R1 but, this time, there is no traffic jam. Also argue that this system optimum is unstable and will revert to the user equilibrium for perfectly informed drivers.

(d) Keeping aside oscillations, derive the following necessary conditions for the percentage α of drivers with active navigation devices (the drivers will always follow the recommendations) and travel-time uncertainties σ_T between the devices for realizing an UE or SO,

$$\text{UE:} \quad (\alpha > P_2^{\text{SO}}) \text{ AND } \sigma_T \ll T_{02} - T_{01} \tag{1}$$

$$\text{SO:} \quad (P_2 < 50\%) \text{ AND } \left(\alpha = \alpha_{\text{SO}} = P_2^{\text{SO}} \left(e^{\frac{T_{02}-T_{01}}{\sigma_T}} + 1\right)\right) \tag{2}$$

where $P_2^{\mathrm{SO}} = (Q_{\mathrm{in}} - C_1^{\mathrm{B}})/Q_{\mathrm{in}} = 10\%$ is the assignment for SO (and also UE), $T_{02} - T_{01} = 50\,\mathrm{s}$ is the time difference for the empty network (and also in the SO). The time uncertainties are modelled with the *Logit* ansatz. For given *ground truth* instantaneous travel times T_1 and T_2, the navigation devices recommend Route R2 with the probability

$$P_2^{\mathrm{nav}} = \frac{1}{1 + \exp\left(\frac{T_1 - T_2}{\sigma_T}\right)},$$

so the percentage of deviating drivers is given by $\alpha P_2^{\mathrm{nav}}$

(e) Finally, derive following necessary condition for the Route 2 to remain uncongested,

$$\alpha \le \alpha_{\max} = P_2^{\max}\left(e^{\frac{T_{02} - T_{01}}{\sigma_T}} + 1\right), \tag{3}$$

with $P_2^{\max} = C_2^{\mathrm{B}}/Q_{\mathrm{in}} = 20\%$, and show that, provided no oscillations build up, no jams on either route are observed if

$$\alpha_{\mathrm{SO}}(\sigma_T) \le \alpha \le \alpha_{\max}(\sigma_T). \tag{4}$$