# Game-theoretic approach to lane-changing in microscopic traffic models

Arne Kesting [a,*], Martin Treiber [a], Dirk Helbing [a,b]

[a] *Technische Universität Dresden, Institute for Transport & Economics, Andreas-Schubert-Strasse 23, D-01062 Dresden, Germany*

[b] *Collegium Budapest – Institute for Advanced Study, Szentháromság u. 2, H-1014 Budapest, Hungary*

**Abstract**

We propose a general scheme to derive lane-changing rules for a wide class of microscopic single-lane traffic models. Both the utility of a given lane and the risk associated with lane changes is determined in terms of longitudinal accelerations calculated with the microscopic traffic models. Thus, anticipative elements and the influence of velocity differences of these longitudinal models are automatically transferred to the lane-changing rules. This allows for a compact and general lane-changing model formulation. An essential ingredient of the proposed lane-changing model is the 'politeness factor' measuring the degree of cooperation or altruism when considering lane changes. Via the politeness, the (dis-)advantages of other drivers associated with a lane change are taken into account. Depending on the value of this parameter, either a local user optimum or system optimum with respect to lane choice is approached. We apply the concept to common car-following models and a cellular automaton. Simulations of symmetric and asymmetric passing rules with the 'intelligent driver model' (IDM) show a good agreement with empirical results such as lane-changing rates and relative lane usages as a function of traffic density.

*Key words:* Micro-simulation; Lane changing; Merging; Multi-lane traffic simulation; Microscopic traffic model

* Corresponding author. Tel. ++49 351 463 36838; fax: ++49 351 463 36809
*Email address:* `kesting@vwi.tu-dresden.de` (Arne Kesting).
*URLs:* `http://www.traffic-simulation.de` (Martin Treiber),
`http://www.helbing.org` (Dirk Helbing).

## 1 Introduction

Freeway lane changing has received increased attention during the last years (Laval and Daganzo, 2005; Hidas, 2005; Coifman et al., 2005; Wei et al., 2000; Brackstone et al., 1998; Nagel et al., 1998; Chang and Kao, 1991). Since lane changing maneuvers often act as initial perturbations, it is crucial to understand their impact on the capacity, stability, and breakdown of traffic flows. Particularly near bottleneck sections such as on-ramps and off-ramps, lane changing is often a significant ingredient to trigger a traffic breakdown (provided that the traffic volume is high) (Laval and Daganzo, 2004). Additionally, the drivers' lane-changing behavior has direct influence on traffic safety. Obviously, a detailed understanding of the lane-changing processes is rather difficult. Operational and tactical layers for mandatory and discretionary merges together with traffic legislation and driving experiences result in a complex perception and decision process of the individual driver (Salvucci and Liu, 2002).

Despite its great significance, lane-changing has by far not been studied as extensively as the longitudinal acceleration and deceleration behavior. One reason for this is the scarcity of reliable data (Hidas and Wagner, 2004; Brackstone and McDonald, 1996). For measuring lane-changes, cross-sectional data from detectors are not sufficient. Recent progress in video tracking methods, however, allows for a collection of high-quality trajectory data from aerial observations (Hoogendoorn et al., 2003). These 2D data will be more and more available for a detailed analysis in the near future.

Within the last years, single-lane car-following models have been successfully applied to describe traffic phenomena (see, e.g. (Helbing, 2001)). Particularly, collective phenomena such as traffic instabilities and the spatiotemporal dynamics of congested traffic states have mainly been accounted for by longitudinal traffic models, describing the acceleration and braking behavior and the appropriate response of the following vehicle. Thus, many observations can be well understood within the scope of single-lane models. But real traffic consists of different types of vehicles, e.g., cars and trucks. Within single-lane models, such heterogeneous traffic can not realistically represented, because the slower vehicles determine the traffic dynamics, while the faster ones are trapped behind them. A realistic description of heterogeneous traffic is, therefore, only possible within a multi-lane simulation framework allowing faster vehicles to improve their (perceived) driving conditions by passing slower vehicles.

The modeling of lane changes is typically considered as a multi-step process. On a *strategic* level, the driver knows about his or her route in a network, which influences the lane choice, e.g., with regard to off-ramps or mandatory merges. In the *tactical* stage, a future lane change is characterized by advance accelerations or decelerations of the driver intending to change, and possibly by cooperation of drivers in the target lane (Hidas, 2005). Finally, in the *operational* stage, one determines if an immediate lane change is both safe and desired. If this is the case, the lane change is performed immediately. The goal of the tactical stage is to favor a positive operational decision. In this contribution, we will present a lane-changing model for microscopic car-following models, which describes the rational *decision* to change lanes and, therefore, only deals with the operational decision process.

Pioneering work dealing with the macroscopic lane-changing process was proposed by Gazis *et al.* (Gazis et al., 1962). Sparmann (Sparmann, 1978) and Gipps (Gipps, 1986) presented lane change decision models to be used for microscopic traffic simulation. These models both distinguish between a safety and an incentive criterion for lane changes. A lane change in the rule-based model of Yang and Koutsopoulos (Yang and Koutsopoulos, 1996) is classified as either mandatory or discretionary. Gap acceptance models are used in order to check whether to accept or reject the available adjacent gap (Halati et al., 1997; Ahmed, 1999; Toledo et al., 2003). Hidas (Hidas, 2005) extends the lane-changing process to a cooperative behavior for merging. For cellular-automata traffic models, several sets of lane-changing rules have been proposed as well (Nagel et al., 1998; Wagner et al., 1997; Knospe et al., 2002).

When considering a lane change, a driver typically makes a trade-off between the expected own advantage and the disadvantage imposed on other drivers. For a driver considering a lane change, the subjective utility of a change increases with the gap to the new leader on the target lane. However, if the velocity of this leader is lower, it may be favorable to stay on the present lane despite of the smaller gap. A criterion for the utility including *both* situations is the difference of the accelerations after and before the lane change. In this work, we therefore propose as utility function considering the difference in vehicle accelerations (or decelerations) after a lane change, calculated with an underlying microscopic longitudinal traffic model. The basic idea of our proposed lane-changing model is to formulate the anticipated advantages and disadvantages of a prospective lane change in terms of single-lane accelerations. In contrast to the classical gap-acceptance approach, a lane-changing decision is based on the *vehicle accelerations of the local traffic situation before and after a considered possible lane change*. This allows for a compact and general model formulation in terms of a 'meta-model' in the sense that it can be applied to any continuous microscopic model, and, with a suitable definition of accelerations, even to cellular automata. Later on, we will discuss this interrelation.

Compared to an explicit lane-changing model, the formulation in terms of accelerations of a longitudinal model has several advantages. First, it is ensured that both longitudinal and lane-changing models are consistent with each other. For example, if the longitudinal model is crash-free, the combined models will be crash-free as well. Second, any complexity of the longitudinal model such as anticipation *automatically* is transfered to a similarly complex lane-changing model. Nevertheless, only one additional parameter is needed. Third, the braking deceleration imposed on the new follower on the target lane to avoid accidents is an obvious measure for the (lack of) safety. Thus, safety and motivational criteria can be formulated in a unified way.

Apart from using accelerations as utility functions, the main novel feature of the proposed lane-changing model consists in taking into account the (dis-) advantage of the followers via a *politeness parameter*. By adjusting this parameter, the motivations for lane-changing can be varied from purely egoistic (where drivers attempt to reach their individual 'user optimum' only (Wardrop, 1952)) to a more altruistic behavior. Particularly, there exists a value where lane changes are only carried out if this increases the combined accelerations of the lane-changing driver and all affected neighbors, i.e., each driver contributes to reach a 'system optimum', at least locally. This strategy can be paraphrased by the acronym *'Minimizing Overall Braking Induced by Lane Changes'* (MOBIL). In the following, we will refer to our concept by this acronym, regardless of the value of the politeness parameter. Notice that all lane-changing models cited before assume egoistic behavior. By the politeness factor, we can model two common lane-changing patterns. First, most drivers do not change lanes for a marginal advantage if this obstructs other drivers. Second, particularly in European countries with asymmetric lane-changing rules, 'pushy' drivers may induce a lane change of a slower driver in front of them in order to be no longer obstructed. In addition, this approach may serve as a basis to generalize game-theoretic considerations for route choices (Selten et al., 2004; Helbing et al., 2005b) to lane choices.

Our paper is organized as follows: In Sec. 2, we formulate the lane-change model MOBIL both for symmetric ('US') and asymmetric ('European') passing rules. The model implementation and its parameters are discussed in Sec. 2.4. In Sec. 3, we apply the general lane-changing rules to simple traffic models and show that the special 'egoistic' case with politeness factor $p = 0$ results in simple gap-acceptance rules as in the previous literature. In Sec. 4, for illustrative reasons, we apply the MOBIL rules and simulate multi-lane traffic in combination with the 'intelligent driver model' (IDM) as underlying longitudinal car-following model (Treiber et al., 2000). We will investigate the lane-changing rate, the velocity-density relation, and the lane usage as a function of traffic density, since these measurable quantities can be compared with empirical data. Finally, in Sec. 5, we will conclude with a discussion of the proposed model, future research directions, and a generalization of the MOBIL concept to other traffic-related decision processes, e.g., when approaching traffic lights.

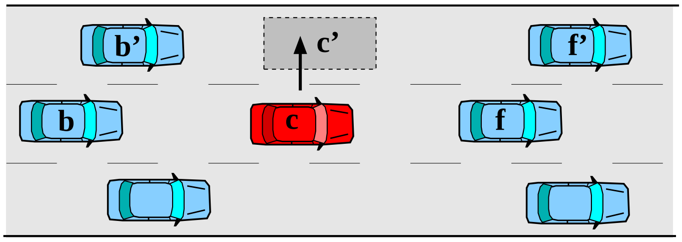



Fig. 1. Sketch of the nearest neighbors of a central vehicle $c$ considering a lane change to the left. The new and old successors are denoted by $n$ and $o$, respectively. Accelerations after a possible change are denoted with a tilde.

## 2 The lane-changing model MOBIL

Most time-continuous microscopic single-lane traffic models describe the motion of single 'driver-vehicle units' $\alpha$ as a function of their own velocity $v_\alpha$, the bumper-to-bumper distance $s_\alpha$ to the front vehicle $(\alpha - 1)$ and the relative velocity $\Delta v_\alpha = v_\alpha - v_{\alpha-1}$. The acceleration of these car-following models is of the general form

$$a_\alpha := \frac{dv_\alpha}{dt} = a(s_\alpha, v_\alpha, \Delta v_\alpha), \tag{1}$$

(Brackstone and McDonald, 1999). Some examples are the Gipps model (Gipps, 1986), the 'optimal velocity model' (Bando et al., 1995), the 'intelligent driver model' (Treiber et al., 2000), or the 'velocity difference model' (Jiang et al., 2001; Helly, 1959). The form of Eq. (1) also covers models with a discontinuous acceleration function such as the Wiedemann model (Wiedemann, 1974). Even some time-discrete models and cellular automata can be treated as shown in Sec. 3 below. Moreover, a generalization to models taking into account more than one predecessor (Belexius, 1968; Lenz et al., 1999; Treiber et al., 2006), or to models with explicit reaction time, is straightforward.

A specific lane change, e.g., from the center lane to the median lane as shown in Fig. 1 depends generally on the two following vehicles on the present and the target lane, respectively. In order to formulate the lane-changing criteria, we use the following notation: For a vehicle $c$ considering a lane change, the successive vehicles on the target and present lane are represented by $n$ and $o$, respectively. The acceleration $a_c$ denotes the acceleration of vehicle $c$ on the actual lane, while $\tilde{a}_c$ refers to the situation on the target lane, i.e., to the new acceleration of vehicle $c$ on the target lane. Likewise, $\tilde{a}_o$ and $\tilde{a}_n$ denote the acceleration of the old and new followers after the lane change of vehicle $c$.

## 2.1 Incentive criterion for symmetric lane-changing rules

Like other lane-changing models (Gipps, 1986), we distinguish between an incentive to change lanes and safety constraints. The *incentive criterion* typically determines if a lane change improves the individual local traffic situation of a driver. In our *ansatz*, we generalize the incentive criterion to include the immediately affected neighbors as well. The *politeness factor* $p$ determines to which degree these vehicles influence the lane-changing decision. For effectively symmetric overtaking rules, we neglect differences between the lanes and propose the following incentive condition for a lane-changing decision of the driver of vehicle $c$:

$$\underbrace{\tilde{a}_c - a_c}_{\text{driver}} + p\Big( \underbrace{\tilde{a}_n - a_n}_{\text{new follower}} + \underbrace{\tilde{a}_o - a_o}_{\text{old follower}} \Big) > 0. \qquad (2)$$

The first two terms denote the advantage (utility) of a possible lane change for the driver him- or herself, where $\tilde{a}_c$ refers to the new acceleration after a prospective lane change for vehicle $c$. The considered lane change is favorable, if the driver can accelerate more, i.e. go faster in the new lane. The third term with the *politeness factor* $p$ is the main innovation in our model. It denotes the total advantage (acceleration gain or loss, if negative) of the two immediately affected neighbors, weighted with $p$. In summary, the incentive criterion is fulfilled if the own advantage (acceleration gain) is higher than the weighted sum of the disadvantages (acceleration losses) of the new and old successors.

Notice that not only the follower in the target lane is taken into account, but also the follower in the old lane who generally takes an advantage of the lane change. Consequently, for sufficiently high values of $p$, 'pushy' successors may induce a lane change, even if this is not favorable for the lane-changing driver him- or herself. The generalization to traffic on more than two lanes per direction is straightforward. If, for a vehicle on a center lane, the incentive criterion is satisfied for both neighboring lanes, the change is performed to the lane where the incentive is larger.

In the symmetric incentive criterion (2), differences between the lanes are neglected. However, by adding a constant bias $\Delta a_{\text{bias}}$ to the right-hand side of Eq. (2), one may distinguish between the left and right neighboring lanes. Thus, a 'keep-right directive' can be reflected by adding a bias term $\Delta a_{\text{bias}}$ to the incentive criterion always in favor of the right lane.

Since the disadvantages of other drivers and the own advantage are balanced *via* the politeness factor $p$, the lane-changing model contains typical strategical features of classical game theory. The value of $p$ can be interpreted as the degree of altruism. It can vary from $p = 0$ (for selfish lane-hoppers) to

$p > 1$ for altruistic drivers who do not change if that would deteriorate the overall traffic situation considering the followers, while they would perform even disadvantageous lane changes if this improves the situation of the followers sufficiently. Notice that, by setting $p < 0$, even malicious drivers could be modeled, who accept own disadvantages in order to thwart others. We found that realistic lane-changing behavior results for politeness parameters in the range $0.2 < p < 1$ (cf. Sec. 4 below). Notice that, in the special case $p = 1$, the incentive criterion simplifies to

$$\tilde{a}_c + \tilde{a}_n + \tilde{a}_o > a_c + a_n + a_o. \tag{3}$$

Thus, lane changes are only performed when they increase the sum of accelerations of all involved vehicles, which corresponds to the concept *'Minimizing Overall Braking Induced by Lane Changes'* (MOBIL) in the ideal sense. In this case, no additional safety constraint is needed since a braking maneuver in order to avoid an accident would be automatically excluded by Eq. (3).

### *2.2 Safety criterion*

In the case $p < 1$, a *safety criterion* must be additionally satisfied for the follower on the target lane to avoid accidents. The safety criterion guarantees that, after the lane change, the deceleration of the successor $\tilde{a}_n$ on the target lane does not exceed a given limit $b_{\text{safe}}$, i.e.,

$$\tilde{a}_n \geq -b_{\text{safe}}. \tag{4}$$

Although formulated as a simple inequality, this condition contains all the information provided by the longitudinal car-following model via the acceleration $\tilde{a}_n$. In particular, if the longitudinal model has a built-in sensitivity with respect to *velocity differences*, this dependence is transfered to the lane-changing decisions. In this way, larger gaps between the following vehicle in the target lane and the own position are required to satisfy the safety constraint, if the following vehicle is faster than the own speed. In contrast, lower values for the gap are allowed if the back vehicle is slower. Moreover, by formulating the criterion in terms of safe braking decelerations of the longitudinal model, crashes due to lane changes are *automatically* excluded as long as the longitudinal model itself guarantees a crash-free dynamics. For realistic longitudinal models, $b_{\text{safe}}$ should be well below the maximum possible deceleration $b_{\text{max}}$ (about $9\,\text{m/s}^2$ on dry road surfaces).

In most European countries, the driving rules for lane usage are restricted by legislation. We will now formulate an asymmetric lane-changing criterion for two-lane freeways and assume, without loss of generality, that the right lane is the default lane, i.e., a 'keep-right' directive is implemented. A reformulation for left-oriented traffic (describing, e.g., traffic rules in the UK) as well as generalizations to more than two lanes are straightforward. Specifically, we assume the following 'European' traffic rules: (i) *Passing rule*: Passing on the right-hand lane is forbidden, unless traffic flow is congested, in which case the symmetric rule (2) applies. We treat any vehicle driving at a velocity below some suitably specified velocity $v_{\text{crit}}$ as driving in congested traffic. (ii) *Lane usage rule*: The right lane is the default lane. The left lane should be only used for the purpose of overtaking.

We have implemented the passing rule by replacing the longitudinal dynamics on the right-hand lane by the condition

$$a_c^{\text{eur}} = \begin{cases} \min(a_c, \tilde{a}_c) & \text{if } v_c > \tilde{v}_{\text{lead}} > v_{\text{crit}}, \\ a_{\text{c}} & \text{otherwise,} \end{cases} \tag{5}$$

where $\tilde{a}_c$ corresponds to the acceleration on the left lane and $\tilde{v}_{\text{lead}}$ denotes the velocity of the front vehicle on the left-hand lane. The passing rule influences the acceleration on the right-hand lane only, (i) if there is no congested traffic ($\tilde{v}_{\text{lead}} > v_{\text{crit}}$), (ii) if the front vehicle on the left-hand lane is slower ($v_c > \tilde{v}_{\text{lead}}$), and (iii) if the acceleration $\tilde{a}_c$ for following this vehicle would be lower than the single-lane acceleration $a_c$ in the actual situation. Notice that the condition $v_c > \tilde{v}_{\text{lead}}$ prevents that vehicles on the right-hand lane brake whenever they are passed.

The 'keep-right' directive is implemented by a constant bias $\Delta a_{\text{bias}}$ (cf. Sec. 2.1). Furthermore, we have neglected the disadvantage of the successor in the right lane in Eq. (2), since the left lane has priority. Explicitly speaking, the resulting asymmetric (European) incentive criterion for lane changes from left (L) to right (R), therefore, reads

$$L \to R: \ \tilde{a}_c^{\text{eur}} - a_c + p\,(\tilde{a}_o - a_o) > -\Delta a_{\text{bias}}. \tag{6}$$

Moreover, the incentive criterion for a lane change from right (R) to left (L) is given by

$$R \to L: \ \tilde{a}_c - a_c^{\text{eur}} + p\,(\tilde{a}_n - a_n) > \Delta a_{\text{bias}}. \tag{7}$$

Neglecting the follower in the right-hand lane leads to a different interpretation of the politeness parameter $p$ than for the symmetric rule. Taking into account only the follower of the faster (left) lane via the politeness factor $p$ applies a selective *'dynamic pressure'* on slow vehicles driving on the left lane in order to let fast vehicles pass on the left lane. For a driver on the right lane, a lane change to the left lane is prevented if a vehicle on the left lane is quickly approaching, reflecting a reasonable driver behavior. Notice that, independently of this, the safety criterion (4) will prevent critical lane changes.

In reality, the decision process for lane-changing under asymmetric driving rules is even more complex than expected according to the driving rules (6) and (7). While one *should* turn to the right-most lane if possible, the passing rule implies the risk of getting stuck behind slower vehicles or trucks, thus, having difficulties to change to the left neighboring lane in order to pass later on. In response to this inconvenience, car drivers anticipate possible hindrances by slower vehicles in the right lane for some time in the future. We model these considerations by evaluating the prospective accelerations $\tilde{a}_\alpha$ occurring in (6), (7) together with the condition (5), assuming a smaller gap than in reality whenever the front vehicle is on the right-hand lane, i.e., in this case we replace $a_\alpha$ in Eq. (1) by

$$a^*_\alpha := a\left(\alpha_s s_\alpha, v_\alpha, \Delta v_\alpha\right). \tag{8}$$

The factor $\alpha_s < 1$ reflects the anticipation of smaller future distances $s^* = \alpha_s s$ to front vehicles on the right-hand lane. Consequently, the situation on, or a change to the right-hand lane is only considered favorable, if the situation on this lane is also favorable for these smaller gaps. Notice that this mechanism is not applied at high traffic densities since, in this case, the symmetric criterion (2) applies.

### *2.4 Discussion of the model implementation and the parameters*

Let us discuss some additional aspects of the implementation and the parameter calibration of the proposed lane-changing model MOBIL. The incentive criterion is evaluated in each numerical update step of the simulation, i.e., the drivers continuously check their incentives. So far, any lane change based on the MOBIL decision to change lanes has been assumed to occur *instantaneously* and in purely lateral direction, although, in reality, the entire lane-changing maneuver takes a finite time. We take care of this fact by introducing a lane-change duration $\tau$ of 3 seconds after each lane change, during which further lane changes are suppressed for that particular vehicle. In addition, we have applied this 'waiting time' also to the new follower to prevent a simultaneous lane change in the opposite direction. We have checked various values for $\tau$ and found that the lane-changing rate essentially becomes independent of $\tau$ for values of $\tau > 2\,\mathrm{s}$.

When evaluating the MOBIL accelerations for the old and new followers, one has, in principle, the freedom to evaluate the accelerations using the own model parameter set or that of the respective successors. Clearly, using the driving parameters of the followers is in line with the reasoning behind MOBIL, although they are not directly observable by the driver initiating a lane change. However, strong clues are given to the driver both by the vehicle type (truck, family car, sports car) and by the past driving style. Therefore, we evaluate all MOBIL accelerations with the model parameters of the respective successors.

The 'ideal' MOBIL strategy corresponding to $p = 1$ has no free parameters and might therefore be considered as a 'minimal model' for lane-changing decisions. However, realistic lane-changing rates require politeness factors $p < 1$. In this case, the lane-changing rules additionally contain the parameter $b_{\text{safe}}$ for the safety criterion, and a finite lane-changing time $\tau$. In Sec. 3, we will show that also the most simple variant of MOBIL characterized by $b_{\text{safe}} = p = 0$ leads to reasonable lane changing behavior. The maximum safe deceleration $b_{\text{safe}}$ prevents accidents even in the case of totally selfish drivers ($p = 0$) as long as its value is not greater than the maximum possible deceleration $b_{\max}$ of the underlying longitudinal model. Increasing the value for $b_{\text{safe}}$ generally leads to stronger perturbations due to individual lane changes. The politeness factor $p$ primarily determines the rate of lane changes (per km and hour) and will be analyzed in more detail in Sec. 4.1.

The lane-changing model for explicitly asymmetric, European driving rules requires additional parameters. The critical velocity $v_{\text{crit}}$ for the crossover from symmetric to European rules is given by legislation. The asymmetry bias $\Delta a_{\text{bias}}$ and the distance scaling factor $\alpha_s$ control the percentage of vehicles using the right-most lane. While $\Delta a_{\text{bias}}$ prevails for low traffic densities, $\alpha_s$ can be calibrated to reproduce the lane-usage inversion for 'European' traffic rules. For very low densities, interactions are not relevant, and the bias $\Delta a_{\text{bias}}$ leads to a predominant usage of the right-most lane. For medium densities, interactions become important, and the asymmetry of the interactions (enforced by the distance scaling factor $\alpha_s$) is the main effect. It leads to the phenomenon of *lane usage inversion* frequently observed on European freeways (Shvetsov and Helbing, 1999; Nagel et al., 1998; Wagner et al., 1997) and is further investigated in Sec. 4.3. For even higher densities, the criterion $v_\alpha < v_{\text{crit}}$ for obstructed traffic is satisfied and symmetric traffic rules are applied, resulting in a more symmetric lane usage.

Due to the weaker acceleration capability and different driving behavior of trucks, their acceleration-related lane-changing parameters need to be smaller compared to those of cars. The fact that most truck drivers will overtake other vehicles even if the velocity difference is rather small and other drivers are obstructed can be modeled by assuming low values of the politeness factor or even $p = 0$. For European driving rules, however, $p > 0$ guarantees a realistic 'dynamic pressure' exerted by fast followers on truck drivers in the fast lane

Table 1
Parameters of the MOBIL model and their reasonable values or ranges for cars and trucks. The most important parameter is the politeness $p$. For $p < 1$, the maximum safe deceleration $b_{\text{safe}}$ serves as additional safety criterion. The value of $b_{\text{safe}}$ is chosen considerably below the physically possible maximum deceleration (about $9\,\text{m/s}^2$). The bias $\Delta a_{\text{bias}}$ models a preferred lane-usage, which often varies between cars and trucks. In the case of asymmetric (European) lane-changing rules, the threshold $v_{\text{crit}}$ distinguishes between congested and free traffic. Optionally, the distance scaling factor $\alpha_s$ can be used to reproduce the effect of lane-inversion. For realistic simulations, the lane-changing time $\tau$ accounts for the finite duration of a lane-changing maneuver. Notice that the parameters depend on the respective longitudinal traffic model. The values listed below are used in combination with the 'intelligent driver model' (IDM) in the simulations of Sec. 4 unless stated otherwise.

| Parameter | Car | Truck |
|---|---|---|
| Politeness Factor $p$ | $0 \ldots 1$ | $0 \ldots 0.3$ |
| Maximum safe deceleration $b_{\text{safe}}$ | 4 $\text{m/s}^2$ | 4 $\text{m/s}^2$ |
| Bias (for right lane) $\Delta a_{\text{bias}}$ | $0 \ldots 0.3$ $\text{m/s}^2$ | 0.3 $\text{m/s}^2$ |
| European Traffic Rules $v_{\text{crit}}$ | 60 km/h | 60 km/h |
| Distance Scaling Factor $\alpha_s$ | 0.2 ... 1 | 1 |
| Lane-changing time $\tau$ | 3 s | 3 s |

to turn back to the right-hand lane as soon as possible.

In Sec. 4, we will investigate the MOBIL parameters by means of simulations. It turns out that the MOBIL model is relatively easy to calibrate, since each MOBIL parameter is responsible essentially for a single effect and has an intuitive interpretation. The reasonable parameter ranges for cars and trucks are listed in Table 1. Notice that lane-changing properties and, consequently, the calibrated values depend on the respective longitudinal traffic model, so the calibration results should not be taken as model-independent quantities.

## 3 Application to simple microscopic traffic models

For reasons of illustration, we will now apply the symmetric incentive criterion (2) with $p = 0$ and the safety criterion (4) to the 'optimal velocity model' (OVM) (Bando et al., 1995) and the cellular automaton Nagel-Schreckenberg model (Nagel and Schreckenberg, 1992). The OVM acceleration equation for a vehicle $\alpha$ can be written in the form

$$a_\alpha(t) = \frac{\mathrm{d}v_\alpha}{\mathrm{d}t} = \frac{V_{\text{opt}}(s_\alpha(t)) - v_\alpha(t)}{\tau}, \tag{9}$$

where $V_{\mathrm{opt}}(s)$ represents the 'optimal velocity function', i.e., the equilibrium velocity for a given spatial vehicle gap $s$. Defining the inverse $s_{\mathrm{opt}}(v)$ of this function, i.e., the equilibrium distance for a given velocity, the safety criterion (4) implies for the new follower $n$ on the target lane a safe distance of

$$\tilde{s}_n > s_{\mathrm{opt}}\left(v_n - \tau b_{\mathrm{safe}}\right), \tag{10}$$

while the incentive criterion (2) implies

$$\left(\frac{\mathrm{d}V_{\mathrm{opt}}(s_c)}{\mathrm{d}s} > 0\right) \quad \text{AND} \quad \left(s_c < \tilde{s}_c\right). \tag{11}$$

As a consequence, the resulting lane-changing rules define a simple gap-acceptance model: The safety criterion is fulfilled, if the gap $\tilde{s}_n$ to the back vehicle on the target lane is larger than the equilibrium gap for the actual velocity $v_n$, reduced by $\tau b_{\mathrm{safe}}$. The incentive criterion is satisfied, if there is an interaction at all ($V'_{\mathrm{opt}}(s) > 0$), and if the gap to the front vehicle $\tilde{s}_c$ on the other lane is larger. Notice that, for typical OVM parameter values such as $\tau = 0.5$ s and $b_{\mathrm{safe}} = 4$ m/s$^2$, the velocity difference $\tau b_{\mathrm{safe}}$ appearing in Eq. (10) is a small quantity.

Now, we will apply the symmetric MOBIL model with $p = 0$ to the deterministic part of the Nagel-Schreckenberg model defined by the update rule

$$v_\alpha(t+1) = \min(v_\alpha + 1, v_0, s_\alpha), \tag{12}$$

where $v_\alpha$ is the velocity of vehicle $\alpha$ in units of 7.5 m/s, $v_0$ the maximum velocity, and $s_\alpha$ the gap measured by the number of empty cells of 7.5 m length. This rule may be interpreted as a discretized version of the car-following equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \min(1, v_0 - v, s - v). \tag{13}$$

Applying the symmetric MOBIL safety and incentive criteria to this leads to the lane-changing rules

$$\tilde{s}_n > v_n - b_{\mathrm{safe}}, \tag{14}$$

and

$$(s_c < v_0) \text{ AND } (s_c < \tilde{s}_c). \tag{15}$$

Remarkably, for $b_{\mathrm{safe}} = 0$, these rules are identical to the rules proposed by (Wagner et al., 1997).

In summary, the MOBIL scheme produces purely gap-oriented lane-changing rules when applied to the OVM and the Nagel-Schreckenberg model, i.e., the required gap sizes depend on the own velocity but not on velocity differences. This is not very realistic, but results from the fact that the underlying longitudinal models do not depend on the velocity difference themselves. In the rest of this paper, we will apply the MOBIL concept to the 'intelligent driver model' (IDM) (Treiber et al., 2000). Both the IDM and the lane-changing rules resulting from it depend on velocity differences.

## 4 Application to multi-lane traffic simulations

Let us now demonstrate the MOBIL concept by means of traffic simulations. Since the rules are formulated in a model-independent way based on longitudinal accelerations, we have to specify the underlying microscopic traffic model. In the following, we use the 'intelligent driver model' (IDM) (Treiber et al., 2000), which is a simple microscopic model with intuitive parameters.

The IDM acceleration of each vehicle $\alpha$ is a continuous function of the velocity $v_\alpha$, the net distance gap $s_\alpha$, and the velocity difference (approaching rate) $\Delta v_\alpha$ to the leading vehicle:

$$\dot{v}_\alpha = a \left[ 1 - \left( \frac{v_\alpha}{v_0} \right)^4 - \left( \frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha} \right)^2 \right]. \tag{16}$$

This expression is an interpolation of the tendency to accelerate with $a_{\text{free}}(v) = a[1 - (v/v_0)^4]$ on a free road and the tendency to brake with deceleration $-b_{\text{int}}(s, v, \Delta v) = -a(s^*/s)^2$, when vehicle $\alpha$ comes too close to the vehicle ahead. The deceleration term depends on the ratio between the effective 'desired minimum gap'

$$s^*(v, \Delta v) = s_0 + vT + \frac{v \Delta v}{2\sqrt{ab}} \tag{17}$$

and the actual gap $s_\alpha$. The minimum distance $s_0$ in congested traffic is significant for low velocities only. The main contribution in stationary traffic is the term $vT$, which corresponds to following the leading vehicle with a constant 'safety' time headway $T$. The last term is only active in non-stationary traffic and implements an accident-free 'intelligent' driving behavior including a braking strategy that, in nearly all situations, limits braking decelerations to the 'comfortable deceleration' $b$. Notice that the IDM guarantees crash-free driving.

The lane-changing behavior does not only depend on the lane-changing model, but also on the heterogeneity of the driver-vehicle parameters. Particularly, for identical driver-vehicle units, a stationary state would be reached soon. To avoid this artefact, we have introduced heterogeneity in the simplest possible way by implementing two types of vehicles. The slower 'trucks' differ only in their reduced desired velocity $v_0 = 85\,\text{km/h}$ compared to the faster 'cars' ($v_0 = 130\,\text{km/h}$). Furthermore, we assumed a truck fraction of 20% in our simulations. We use the following IDM parameters: The time headway is set to

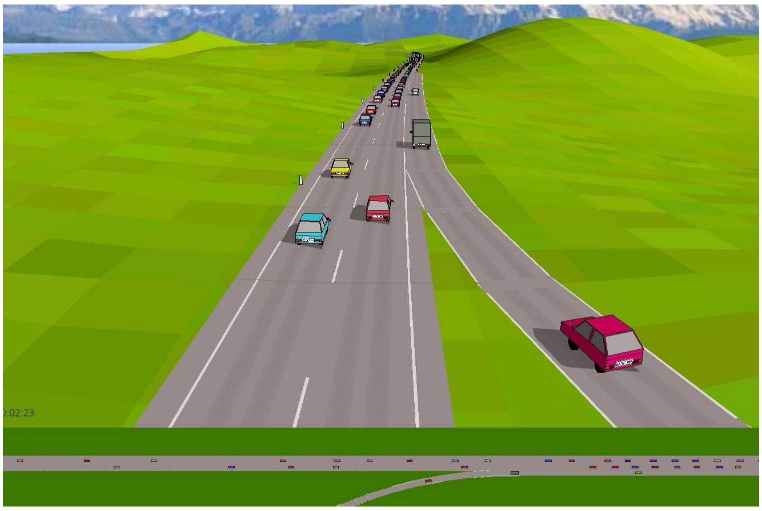


Fig. 2. Screenshot of the visualization front-end of the multi-lane traffic simulator developed by the authors. The website `http://www.traffic-simulation.de` provides an interactive simulation of the 'intelligent driver model' (IDM) in combination with the lane-changing model MOBIL.

$T = 1.2$ s, the maximum acceleration to $a = 1$ m/s$^2$, the desired deceleration to $b = 2$ m/s$^2$, and the minimum distance in a traffic jam to $s_0 = 2$ m. Furthermore, the vehicle length (which is not a model parameter) is assumed to be 4.3 m for cars and 12 m for trucks.

When testing lane-changing models for their plausibility, a dynamic visualization in 2D and 3D turned out to be an important tool (Fig. 2), as the lane-changing maneuvers are rather complex. Visualization is also useful to assess some aspects of the MOBIL model such as the 'dynamic pressure' exerted by 'pushy' drivers on slower leaders. Since this effect has little influence on capacity, stability, or other measurable properties, it cannot be tested by other means.

The simulation set-up is as follows: We have simulated a two-lane road section of 10 km length with either open or periodic boundary conditions. For periodic boundary conditions, the traffic density $\rho$ can be controlled directly. We have run multiple simulations varying $\rho$ in a range from 1 to 40 vehicles/km/lane. For open boundary conditions, the inflow $Q_{\text{in}}$ at the upstream boundary is a natural control parameter. Furthermore, for simulations scenarios with open boundaries, we have assumed an on-ramp of merging length 300 m located at location $x = 7.5$ km, which serves as bottleneck. We have run multiple simulations with inflows from 100 vehicles/h/lane up to 1800 vehicles/h/lane and a constant ramp flow of $Q_{\text{rmp}} = 500$ vehicles/h.

In the subsequent sections, we investigate measurable traffic quantities such as the lane-changing rate, the velocity-density relation, and the lane usage as a function of the traffic density $\rho$. The two latter quantities are easily

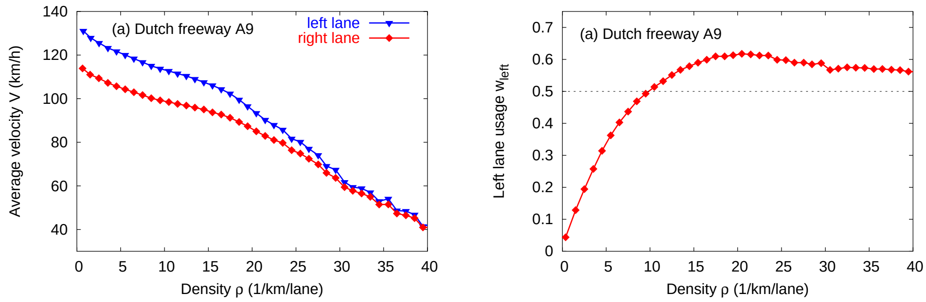



Fig. 3. Mean values of empirical data from the Dutch two-lane freeway A9 measured by double-loop detectors. (a) Relation between mean velocities and lane-averaged density. The lane-averaged velocities synchronize for densities $\rho \gtrsim 30$ /km/lane. (b) Usage of the left lane as a function of density. For medium densities $10 < \rho < 30$ vehicles/km/lane, the data show a distinct lane-inversion, i.e., a higher usage of the faster left lane. The 1-min data from various cross sections on several days within a range of 2 vehicles per kilometer per lane were averaged to reduce the statistical variation.

measurable by double-loop detectors. For illustrative reasons, empirical data from the Dutch two-lane freeway A9 are shown in Fig. 3. In contrast, the direct measurement of the lane-changing events to determine the lane-changing rate is more difficult, which results in a lack of reliable data (Hidas and Wagner, 2004; Brackstone and McDonald, 1996).

### *4.1 Lane-changing rate*

The lane-changing rate measures the performed lane changes per length and time. In the literature, there are only a few empirical studies available. Sparmann investigated the lane-change usage and lane-changing rates on a German two-lane *autobahn* (Sparmann, 1978). Data for a British motorway were presented by Yousif and Hunt (Yousif and Hunt, 1995). According to these measurements, the lane-changing rate has a distinct maximum of approximately 500 to 600 lane changes per hour and km for medium densities of approximately 15 vehicles/km/lane corresponding to flows of about 1000 vehicles/h/lane. Notice that, even for very large traffic densities, the lane-changing rate does not drop to zero. There are still about 100 lane changes per hour and kilometer.

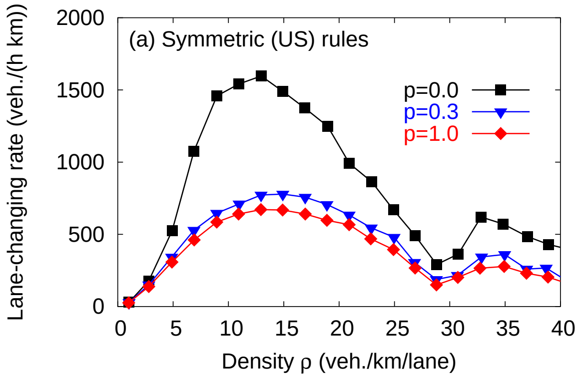
A robust way to measure the lane-changing rate as a function of the traffic density in the simulation both for periodic and open boundary conditions is the following: The road is divided into subsections, e.g., of length $\Delta x = 1$ km, and time is divided into time intervals, whose durations are, e.g., $\Delta t = 1$ min. For each spatiotemporal element $\Delta x \Delta t$ obtained in this way, the number $n$ of lane changes and the average density $\rho$ is determined. The lane-changing rate is then given by

$$r(\rho) = \frac{n}{\Delta x \Delta t}. \tag{18}$$

In the following, we consider the subsection defined by $6 \leq x \leq 7$ km and classify the densities into intervals of width $\Delta\rho = 2$ vehicles per kilometer per lane. Finally, we average over all lane change rates belonging to the same density interval. Taking different values of $\Delta x$, $\Delta t$, or $\Delta\rho$ do not change the results significantly.

In Fig. 4, the averaged lane-changing rates of an open system are shown both for symmetric and asymmetric rules for various values of the politeness factor $p$ for cars, which is the most important parameter for the lane-changing rate. For trucks, the value $p = 0$ has been kept constant. The curves show the following characteristics: (i) The lane-changing rate increases for small densities $1 < \rho < 10$ /km approximately linearly. A more detailed analysis for traffic densities $\rho < 5$ vehicles/km/lane has revealed a quadratic slope as expected from analytical considerations. (ii) The maximum lane-changing rate is reached for medium densities in the range $10 < \rho < 20$ /km. The peak is broad and shows a symmetric, parabolic shape in the case of 'US' overtaking rules. For European rules, only the case $p = 0$ shows a parabolic shape around the maximum, while the peak is deformed towards lower densities for $p > 0$. (iii) The peak value depends strongly on the value of the politeness parameter $p$. For $p = 0$, the maximum lane-changing rate is about 1600/h/km, while a positive value $p > 0$ reduces the maximum significantly. For $p = 0.3$ ($p = 1$), the maximum lane-changing rate is approximately 700 vehicles/h/km both for symmetric and asymmetric rules in agreement with the empirical evidence (Sparmann, 1978). (iv) With increasing density, velocity differences between the lanes diminish (cf. the following section). The lane-changing rates decrease and show a local minimum for dense traffic ($\rho \approx 30$ vehicles/km/lane), when changing lanes is no more profitable or possible due to a lack of suitable gaps. This could be attributed to a 'moving like a solid block' effect proposed in (Helbing and Huberman, 1998). (v) In the congested traffic regime for densities $\rho > 30$ vehicles/km/lane, the lane-changing rates become essentially constant with values between 100 and 400 lane-changes per hour and kilometer depending on $p$.

The lane-changing rates in Fig. 4 show the same characteristics as the empirical observations. The maximum lane-changing rate is predominantly determined by the politeness factor and, therefore, can be easily calibrated. Values of $p \approx 0.3$ are appropriate for realistic lane-changing rates, while $p = 0$ implies much too high lane-changing rates. Of course, one may consider to distribute the value of $p$ in a microscopic simulation to represent different lane-changing behavior among the driver-vehicle-units. Furthermore, we have simulated a system with periodic boundary conditions (with identical values for $p$) and found no qualitative difference in the resulting lane-changing rates as a func-



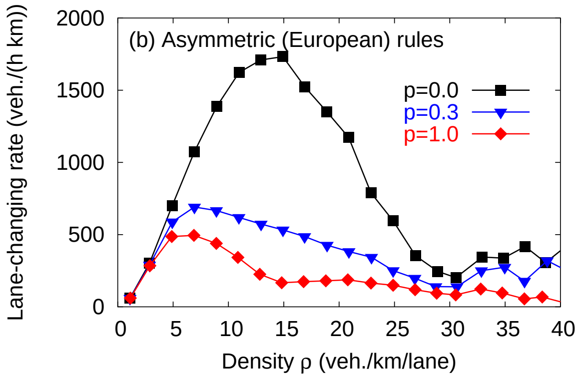




Fig. 4. Lane-changing rates as a function of traffic density (a) for symmetric ('US') lane-changing rules and (b) for asymmetric ('European') rules. The lane-changing rate is mainly determined by the politeness factor $p$. The diagrams show several simulation runs for different values of $p$ for cars. We used a road section of $\Delta x = 1\,\mathrm{km}$ and time intervals of $\Delta t = 1\,\mathrm{min}$ to measure and average the lane-changing rate.

tion of the density.

### *4.2 Velocity-density relation and velocity synchronization*

A consequence of explicitly asymmetric (European) lane-changing rules is a difference of average velocities in different lanes for free traffic. For densities $\rho \gtrsim 30\,\mathrm{vehicles/km/lane}$, however, the velocities synchronize (Helbing, 1997; Helbing and Huberman, 1998), see also Fig. 3(a) for a Dutch freeway. To enable a direct comparison with the data collected from double-loop detectors, we have applied the same data aggregation technique by introducing 'virtual detectors' with 1-min sampling intervals mimicking real-world double-loop detector measurements. We have recorded traffic flow $Q_i$ and total flow $Q = \sum_{i=1}^{L} Q_i$ for a road consisting of $L = 2$ lanes. Furthermore, for each sample interval, we have determined arithmetic velocity averages $V_i$ and the average velocity $V = \sum_{i=1}^{L}(Q_i V_i)/Q$ over all lanes. The density was calculated by the hydrodynamic relation $Q = \rho V$. To facilitate a comparison of empirical and simulation data, we have suppressed the fluctuations in the velocities occurring both in the 1-min data of the real and 'virtual' detectors by averaging over all data belonging to the same density class (of class width $\Delta\rho = 2$ /km/lane).

Figure 5 shows the simulation results for an open system with a detector located at $x = 6$ km for symmetric and asymmetric lane-changing rules. We have varied the politeness factor $p$ of cars. The bias is kept constant ($\Delta a_{\text{bias}} = 0.3$) for cars and trucks for asymmetric rules, while the bias is only applied to trucks in the case of 'US' rules. Therefore, the initially equally distributed trucks are mostly found on the right-most lane. The separation results in a different velocity-density relation for the fast (left) lane and the right lane. The velocity differences decrease with increasing traffic density. For $\rho > 25$ vehicles/km/lane, the mean velocities in adjacent lanes are primarily synchronized in agreement with the empirically observed phenomenon. We have also varied the distance-scaling factor $\alpha_s$ of cars for asymmetric rules. Interestingly, the variation of the parameters $\alpha_s$ and $p$ does not change the shape of the velocity-density relation. Qualitatively, the same behavior as for an open system has also been observed for periodic boundary conditions. Notice that, for symmetric lane-changing rules without any bias, the velocity is basically synchronized in all lanes for all densities due to the lack of any lane preference.

### *4.3 Lane usage and lane inversion*

Another quantity directly available from induction loop measurements is the *lane usage* as a function of traffic density. As the density increases, the percentage of traffic in the right-most lane decreases, while the percentage of traffic in the other lanes increases (May, 1990; Hall and Lam, 1988; Michalopoulos et al., 1984). A frequently observed phenomenon in countries, where regulations prohibit passing on the right is *lane usage inversion*. Empirical detector data for European freeways (Sparmann, 1978; Yousif and Hunt, 1995; Shvetsov and Helbing, 1999) show that, for low densities, vehicles predominantly use the shoulder lane according to the 'keep right' rule. If the traffic density increases, interactions lead to an increasing use of the passing lane characterized by higher velocities. The effect of lane inversion is observed for medium densities of $10 \leq \rho \leq 30$ vehicles/km/lane with a maximum usage of approximately 60% of the faster lane as shown for example in Fig. 3(b). At higher traffic densities, i.e., under congested conditions, all lanes tend towards the same average occupancy and speed (Hall and Lam, 1988). This summarizes our knowledge of the empirical data.

In our computer simulations, the lane usage $w_i = Q_i/Q$ on lane $i$ is also defined by the relative contribution $Q_i$ of the lane $i$ to the overall traffic flow $Q$. Again, we have used the concept of 'virtual detectors' and the same data

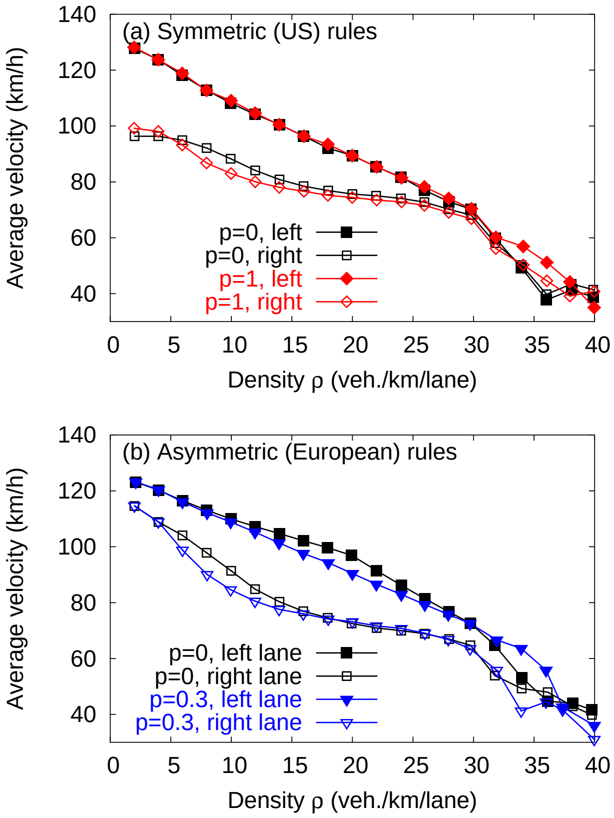



Fig. 5. Velocity-density relation determined from simulations with (a) symmetric lane-changing rules and (b) for asymmetric rules and open boundaries. The velocity and the density are measured by a 'virtual detector' located at $x = 6\,\text{km}$ mimicking the real-world measurement process. The 1-min data within a range of 2 vehicles per kilometer per lane were averaged to reduce the statistical variation.

aggregation as described in the previous subsection. Figure 6 shows the left lane usage as a function of the lane-averaged traffic density $\rho = Q/(LV)$ for the asymmetric MOBIL model. We have varied the politeness factor $p$ and the distance-scaling parameter $\alpha_s$ for cars.

In the case of asymmetric lane-changing rules, the right lane is predominantly used for small densities in agreement with the 'lane-usage rule'. In the high density regime, traffic is nearly equally distributed over all lanes (showing only a weak static effect of lane-inversion). However, for the intermediate density regime $5 < \rho < 25\,\text{vehicles/km/lane}$ and a distance-scaling value of $\alpha_s < 1$, the faster left lane is significantly more occupied than the right lane. The scaling-distance factor, therefore, leads to a distinct lane-inversion effect. Politeness factors $p > 0$ lead to a slower increase of the lane usage $w_{\text{left}}$ of the left lane with increasing traffic density, because slow vehicles avoid the fast lane if it is used by faster vehicles. Basically, the simulation results agree semiquantitatively with the data shown in Fig. 3(b). Again, there is no qualitative difference between simulation scenarios with periodic and open boundary conditions.

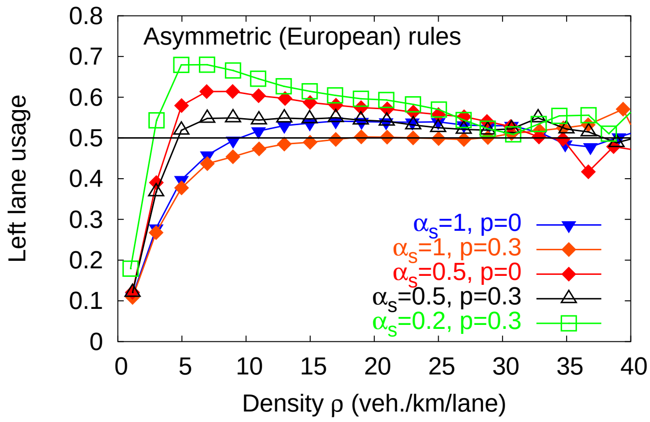



Fig. 6. Lane usage of the fast (left) lane as a function of traffic density for asymmetric rules of a system with open boundary conditions. The 1-min 'virtual detector' data within a range of 2 vehicles per kilometer per lane were averaged to reduce the statistical variation.

The lane inversion could be attributed firstly to a static effect, because the right lane is used more by trucks with a larger vehicle length. Additionally, however, there is a a dynamic effect: Drivers try to avoid travelling behind slower-moving vehicles. Therefore, they avoid changing to the right-most lane, in order not to get stuck behind a slower vehicle.

## 5 Conclusions and outlook

We have presented the concept MOBIL defining lane-changing models for a broad class of single-lane microscopic traffic models, including car-following models and cellular automata. The primary assumption behind MOBIL is to measure both the attractiveness of a given lane and the risk associated with lane changes in terms of accelerations. This means, both the incentive and the safety criteria can be expressed in terms of the acceleration function of the underlying single-lane microscopic model. As a consequence, the properties of the longitudinal traffic model (e.g. any dependence on relative velocities or the exclusion of accidents) are transfered to the lane-change behavior. While simple, gap-oriented microscopic traffic models such as the 'optimal velocity model' (OVM) (Bando et al., 1995) lead to simple gap-acceptance lane-changing models, more elaborate micro-models including anticipation effects and a sensitivity to velocity differences lead to more differentiated lane-changing criteria. When applying the MOBIL model to the 'intelligent driver model' (IDM), it turns out that the parameters can be easily calibrated and the predictions of the resulting multi-lane model agree qualitatively well with the empirical data already for the consideration of only two types of vehicles, cars and trucks. Further improvements can be reached by implementing the full heterogeneity of parameters in real traffic.

The MOBIL concept has few and intuitive parameters. As an essential ingredient, our model takes into account other drivers via a *politeness factor* $p$ characterizing the degree of cooperativeness among drivers, and, thereby, introduces game-theoretical aspects into lane-changing decisions. The politeness factor has, therefore, implications for traffic optimization as well. In the context of route choice decisions in traffic networks, this suggests to transfer the experimental results for the 'route choice game' (Selten et al., 2004; Helbing et al., 2005b) to a 'lane choice game', since MOBIL generalizes this idea to more microscopic space and time scales: Actions corresponding to $p = 0$ lead to the 'user equilibrium', while fully cooperative drivers ($p = 1$) approach a 'system optimum' by *minimizing the overall braking decelerations* (of all involved vehicles) *by lane changes* (MOBIL), at least locally. Moreover, with the increasing availability of trajectory data (Hoogendoorn et al., 2003), accelerations and thus the politeness factor may be directly determined from empirical data, allowing for game-theoretical investigations for real-life situations rather than experimental setups.

The decision process behind MOBIL can be applied to both discretionary and mandatory lane changes. In the latter case, the position of the last possible change (e.g., the end of the acceleration lane at on-ramps) is modelled by a standing 'virtual' vehicle. Thus, the 'pressure' to change to another lane automatically increases when approaching such a mandatory merging point.

Extensions of the proposed acceleration-based concept to other traffic-related decision processes are possible as well. For example, when approaching a traffic light that switches from green to yellow, one has to decide whether to stop in front of the signal, or to pass it. In the framework of MOBIL, the 'stop' decision will be based on the safe braking deceleration $b_{\mathrm{safe}}$. Similar considerations apply when deciding whether it is safe enough to cross an unsignalized intersection (Helbing et al., 2005a), to turn into another road in a 'yield situation', or to start an overtaking maneuver on the opposite lane of a two-way rural road.

Finally, we emphasize that MOBIL is meant to represent only the last 'operational' decision whether to immediately perform a lane change or not. In reality, a lane-changing decision also includes strategical and tactical aspects in preparation for this final step, which are relevant particularly for congested traffic and for mandatory lane changes. For example, tactical behavior may involve accelerations (or decelerations) of the own vehicle or of vehicles on the target lane in preparation for a lane change. This 'longitudinal-transverse coupling' will be the subject of a forthcoming paper.

**Acknowledgments:**

The authors would like to thank for partial support by the Volkswagen AG within the BMBF project INVENT.

## References

Ahmed, K. I., 1999. Modeling drivers' acceleration and lane change behavior. Ph.D. thesis, Massachusetts Institute of Technology.

Bando, M., Hasebe, K., Nakayama, A., Shibata, A., Sugiyama, Y., 1995. Dynamical model of traffic congestion and numerical simulation. Phys. Rev. E 51, 1035–1042.

Belexius, S., 1968. An extended model for car-following. Transportation Research 2 (1), 13–21.

Brackstone, M., McDonald, M., 1996. The microscopic modelling of traffic flow: Weaknesses and potential developments. In: Wolf, D. E., Schreckenberg, M., Bachem, A. (Eds.), Traffic and Granular Flow. World Scientific, Singapore, pp. 151–165.

Brackstone, M., McDonald, M., 1999. Car-following: a historical review. Transportation Research F 2, 181–196.

Brackstone, M., McDonald, M., Wu, J., 1998. Lane changing on the motorway: Factors affecting its occurrence, and their implications. In: Proc. of the 9th Int. Conf. on Road Transportation Information and Control. Conference Publication No. 454, London, pp. 160–164.

Chang, G.-L., Kao, Y.-M., 1991. An empirical investigation of macroscopic lane-changing characteristics on uncongested multilane freeways. Transportation Research A 25A (6), 375–389.

Coifman, B., Krishnamurthy, S., Wang, X., 2005. Lane-changing maneuvers consuming freeway capacity. In: Hoogendoorn, S., Luding, S., Bovy, P., Schreckenberg, M., Wolf, D. (Eds.), Traffic and Granular Flow '03. Springer, Berlin, pp. 3–14.

Gazis, D. C., Herman, R., Weiss, G. H., 1962. Density oscillations between lanes of a multilane highway. Operations Research 10, 658–667.

Gipps, P. G., 1986. A model for the structure of lane-changing decisions. Transportation Research 20B, 403–414.

Halati, A., Lieu, H., Walker, S., 1997. Corsim - corridor traffic simulation model. In: Proceedings of the Traffic Congestion and Traffic Safety in the 21th Century Conference. pp. 570–576.

Hall, F., Lam, T., 1988. The characteristics of congested flow on a freeway across lanes, space, and time. Transportation Research A 22, 45–56.

Helbing, D., 1997. Empirical traffic data and their implications for traffic modelling. Phys. Rev. E 55, R25–R28.

Helbing, D., 2001. Traffic and related self-driven many-particle systems. Reviews of Modern Physics 73, 1067–1141.

Helbing, D., Huberman, B. A., 1998. Coherent moving states in highway traffic. Nature 396, 738–740.

Helbing, D., Jiang, R., Treiber, M., 2005a. Analytical investigation of oscillations in intersecting flows of pedestrian and vehicle traffic. Phys. Rev. E 72, 046130.

Helbing, D., Schönhof, M., Stark, H.-U., Hołyst, J. A., 2005b. How individuals learn to take turns: Emergence of alternating cooperation in a congestion game and the prisoner's dilemma. Advances in Complex Systems 8 (1), 87–116.

Helly, W., 1959. Simulation of bottlenecks in single lane traffic flow. In: General Motors Research Loratories (Ed.), Proceedings of the Symposium on the Theory of Traffic Flow. Elsevier, New York, pp. 207–238.

Hidas, P., 2005. Modelling vehicle interactions in microscopic traffic simulation of merging and weaving. Transportation Research C 13, 37–62.

Hidas, P., Wagner, P., 2004. Review of data collection methods for microscopic traffic simulation. WCTR, Istanbul, Turkey.

Hoogendoorn, S. P., Zuylen, H. J. V., Schreuder, M., Gorte, B., Vosselman, G., 2003. Microscopic traffic data collection by remote sensing. Transportation Research Record 1855, 121–128.

Jiang, R., Wu, Q., Zhu, Z., 2001. Full velocity difference model for a car-following theory. Phys. Rev. E 64, 017101.

Knospe, W., Santen, L., Schadschneider, A., Schreckenberg, M., apr 2002. A realistic two-lane traffic model for highway traffic. Journal of Physics A 35, 3369–3388.

Laval, J. A., Daganzo, C. F., 2004. Multi-lane hybrid traffic flow model: Quantifying the impacts of lane-changing maneuvers on traffic flow. Institute of Transportation Studies. Working Papers. Paper UCB-ITS-WP-2004-1.

Laval, J. A., Daganzo, C. F., 2005. Lane-changing in traffic streams. Transportation Research Part B, in print.

Lenz, H., Wagner, C., Sollacher, R., 1999. Multi-anticipative car-following model. European Physical Journal B 7, 331–335.

May, A., 1990. Traffic Flow Fundamentals. Prentice-Hall, Eaglewood Cliffs, N.Y.

Michalopoulos, P. G., Beskos, D. E., Yamauchi, Y., 1984. Multilane traffic flow dynamics: Some macroscopic considerations. Transportation Research B 18 (4/5), 377–395.

Nagel, K., Schreckenberg, M., 1992. A cellular automaton modell for freeway traffic. J. Phys. I France 2, 2221–2229.

Nagel, K., Wolf, D., Wagner, P., Simon, P., 1998. Two-lane traffic rules for cellular automata: A systematic approach. Phys. Rev. E 58, 1425–1437.

Salvucci, D. D., Liu, A., june 2002. The time course of a lane change: Driver control and eye-movement behavior. Transportation Research Part F 5 (2), 123–132.

Selten, R., Schreckenberg, M., Pitz, T., Chmura, T., Kube, S., Hafstein, S. F., Chrobok, R., Pottmeier, A., Wahle, J., 2004. Experimental investigation of day-to-day route choice-behaviour and simulation of autobahn traffic in north rhine-westphalia. In: Schreckenberg, M., Selten, R. (Eds.), Human Behaviour and Traffic Networks. Springer, Berlin, pp. 1–21.

Shvetsov, V., Helbing, D., 1999. Macroscopic dynamics of multi-lane traffic. Phys. Rev. E 59, 6328–6339.

Sparmann, U., 1978. Spurwechselvorgänge auf zweispurigen BAB-Richtungsfahrbahnen. In: Forschung, Straßenbau und Straßenverkehrstechnik. Vol. 263. Bundesminister für Verkehr, Bonn-Bad Godesberg.

Toledo, T., Koutsopoulos, H. N., Ben-Akiva, M. E., 2003. Modeling integrated lane-changing behavior. Transportation Research Record 1857, 30–38.

Treiber, M., Hennecke, A., Helbing, D., 2000. Congested traffic states in empirical observations and microscopic simulations. Phys. Rev. E 62, 1805–1824.

Treiber, M., Kesting, A., Helbing, D., 2006. Delays, inaccuracies and anticipation in microscopic traffic models. Physica A 360, 71–88.

Wagner, C., Nagel, K., Wolf, D., 1997. Lane-changing rules in two lane traffic simulation using cellular automata. Physica A 234, 687.

Wardrop, J. G., 1952. Some theoretical aspects of road traffic researc. In: Proceedings of the Institution of Civil Engineers II. Vol. 1. pp. 325–378.

Wei, H., Lee, J., Li, Q., Li, C. J., 2000. Observation-based lane-vehicle-assignment hierarchy for microscopic simulation on an urban street network. Transportation Research Record 1710, 96–103.

Wiedemann, R., 1974. Simulation des Straßenverkehrsflußes. Vol. 8 of Schriftenreihe des IfV. Institut für Verkehrswesen, Universität Karlsruhe.

Yang, Q., Koutsopoulos, H. N., 1996. A microscopic traffic simulator for evaluation of dynamic traffic management systems. Transportation Research C 4 (3), 113.

Yousif, S., Hunt, J., December 1995. Modelling lane utilization on British dual-carriageway roads: effects on lane-changing. Traffic Engineering + Control 36 (12), 680 – 687.