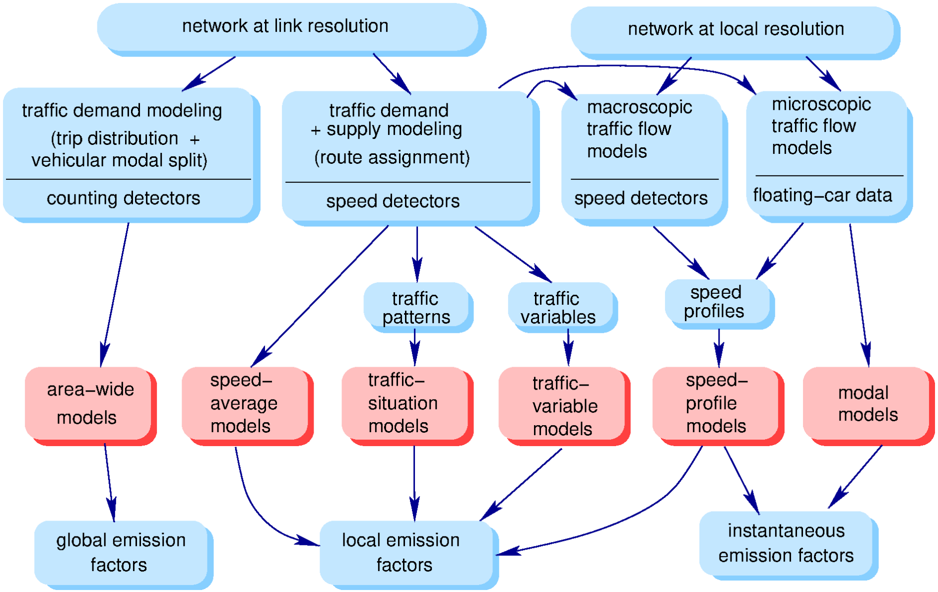
Lecture 12: Fuel, Energy Demand, and Emissions

12.1 General Problem Setting

- Models for fuel consumption, CO_2, and other emissions (NO_x, particulate matter) have the same structure, so all **emission factors** can be discussed together
- Strict proportionality between fuel consumption and CO_2 emissions:
  - Gasoline (98 ROZ): 2.39 kg CO_2/liter
  - Diesel fuel: 2.69 kg CO_2/liter
  - The difference is mainly due to the different specific masses. Essentially, one carbon atom (12 au) produces one CO_2 molecule (44 au), so the mass ratio is about 44/12
- The output can be either **local emission factors** (per distance, e.g.liters/100 km) or **instantaneous factors**, e.g., liters/h. (Always use SI in simulations!)
- Input may be on the link level (not considered here) or local level
- As for traffic flow dynamics, local fuel/emission models can be **macroscopic** or **microscopic**
- We will concentrate on microscopic models

Fuel/emissions model overview



12.2 Speed-Profile Emission Models

The input of **speed-profile models**, also known as **cycle-variable models**, are speed profiles of single vehicles from floating-car data, trajectory data, test cycles, or by a microscopic traffic flow simulation together with vehicle attributes. The output are emissions/emission factors during the duration of the speed profile

- In contrast to modal models, the speed profile is not used directly but aggregated into several **speed profile factors** $\boldsymbol{x}$
- Most approaches use multivariate linear models for estimating the instantaneous emission vector $\boldsymbol{e}$:

$$\boldsymbol{e} = \mathbf{L} \cdot \boldsymbol{x}$$

- The matrix components L_{nm} (to be calibrated) describe the influence of speed profile factor m on emission type n
- Speed-profile factors x_m: fraction of time in a speed class, acceleration standard deviation, ...
- Emission factors: e_n: CO_2, NO_x, PM, ...

Some speed-profile factors

Factor	Effect on CO_2 emissions
Constant of value 1	intercept (+++)
Fraction of time in speed class 0-25 km/h	++
Fraction of time in speed class 50-75 km/h	– –
Fraction of time in speed class 75-100 km/h	–
Fraction of time in speed class $> 125\,\mathrm{km/h}$	++
Standard deviation of speed	+
Average and standard deviation of acceleration	+
Average and standard deviation of deceleration	–
Frequency of acceleration-deceleration cycles	+
Fraction of time the vehicle is standing	+
Fraction of time the vehicle needs power near its maximum power	++
Fraction of road gradients greater than 5 %	+
Engine speed (crankshaft revolution rate) 1 000 - 2 000 rpm	– –
Engine speed (crankshaft revolution rate) $>$ 3 500 rpm	++

12.3 Modal Emission Models

As speed-profile models, **modal emission models** make use of trajectory information but they use them instantaneously and directly: the emission *rate* is an instantaneous function of the **mode** of vehicle operation: speed, acceleration, road gradient etc

- In the more refined modal models, the vehicle operation mode is complemented by a **characteristic map** describing the instantaneous operating mode of the engine in form of fuel and emission rates.
- Depending on the model complexity, further input is necessary including altitude, air temperature, and variables related to the engine history (e.g., engine temperature)
- Perfectly suited to microscopic traffic flow models
- Output: The *rates* $\dot{e}_n$ of the emission factors ($\dot{e}_1$: fuel consumption/CO_2 emission rate, ...)
- Two conceptional types: **phenomenological** models and **physics-based** models

Variants of modal emission models

Phenomenological models

- **Statistical modal models** are based on *regression* including phenomenological interaction terms as factors. For example, for $\dot{C} = \dot{e}_1$

$$\dot{C} = \max\left(0,\ \beta_0 + \beta_1 v + \beta_2 v^2 + \beta_3 v^3 + \beta_4 v\dot{v} + \beta_5 v^2 \dot{v} + ...\right).$$

- **Map-based models**. These are based on **lookup-tables** obtained from real driving experiments (f: engine speed):

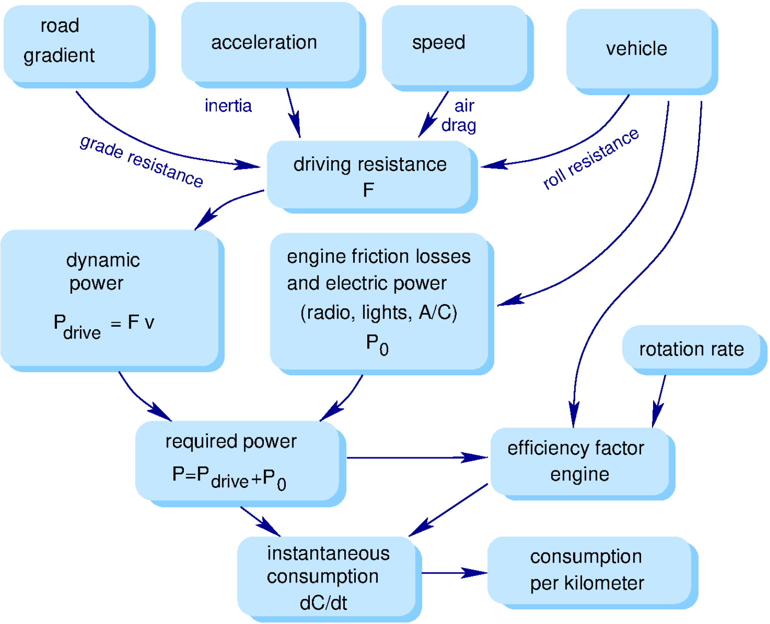
$$\dot{C} = f(v, \dot{v}, f)$$

  - Parameter-free but the driving experiments to generate the lookup tables are very cumpersome
  - No transfer/generalisation ability. Each vehicle/engine combination needs its own lookup table

Physics-based models are based on *first principles* $\Rightarrow$ perfect generalisation ability, even to battery-electric vehicles. Two types:

- purely analytical
- hybrid with an **engine characteristic map**: lookup table generated on test benches

12.4 A Physics-Based Modal Consumption Model



Driving resistance

$$F(v, \dot{v}, \phi) = m_{\text{dyn}}\dot{v} + (\mu + \phi)mg + \frac{1}{2}c_d\rho A v^2$$

- Inertial force $m_{\text{dyn}}\dot{v}$ with the **dynamical mass** m_{dyn} (static mass m + twice the ratio between rotational energy of all rotating parts and v^2, can be up to $m_{\text{dyn}} = 1.5\,m$ for the first gear)
- Solid-state friction force $mg\mu$ ($g = 9.81\,\mathrm{m/s^2}$, friction coefficient $\mu \approx 0.015$)
- Gravitational force of sloping roads $mg\phi$ with the uphill gradient ϕ (an uphill/downhill road gradient of $10\,\%$ will mean $\phi = \pm 0.1$)
- Wind drag $\frac{1}{2}c_d\rho A v^2$
  - c_d: drag coefficient (about 0.3 for normal cars)
  - ρ: air density (about $1.3\,\mathrm{kg/m^3}$ at sea level)
  - A: frontal cross section (about $2\,\mathrm{m^2}$)

Engine/power management

Instantaneous overall power demand:

$$P_{\text{inst}} = P_{\text{drive}} + P_0 = F\,v + P_0$$

- $P_{\text{drive}} = F\,v$: power to overcome the driving resistance
- P_0: power to drive all the electric appliances and to overcome internal engine friction

Several power management options:

- Old vehicles without overrun fuel cutoff:

$$P = P_0 + \max(P_{\text{drive}}, 0)$$

- Contemporary internal combustion vehicles (ICVs) with overrun fuel cutoff:

$$P = \max(P_0 + P_{\text{drive}}, 0) = \max(P_{\text{inst}}, 0)$$

- Vehicles with recuperation ability including battery-electric vehicles (BEV): see Section 12.5

Questions

? Without engaged gear (engine idling, P_0 taken from the starter or driving battery) and without braking, a car just starts to roll on a gentle downhill slope of 1.5 % and reaches a very slow terminal speed. Determine the friction coefficient μ

! With disengaged clutch and no brakes, no driving resistance can be sustained, so $F = 0$. After reaching the slow terminal speed, $\dot{v} = 0$ and the air-drag term is negligible (because of the slow speed). So, with $\phi = -0.015$, we have $\mu + \phi = 0$ or $\mu = -\phi = 0.015$

? On a 3.5 % downhill slope, the same idling car reaches a terminal speed of 108 km/h with no brakes. Estimate its c_d value for $m = 1\,600\,\text{kg}$ and $\rho A = 2\,\text{kg/m}$

! We still have $F = 0$ and $\dot{v} = 0$, so solving the driving resistance equation for c_d, we obtain

$$c_d = \frac{-2(\mu + \phi)mg}{\rho A v^2} = \frac{0.04 * 1\,600 * 9.81}{2 * 30^2} = 0.33$$

? How many kWh mechanical energy are needed for driving 100 km at a constant speed of 100 km/h on a level road? (parameters as above and $P_0 = 2\,\text{kW}$)

! Just insert in any of the three power formulas (neither overrun fuel cutoff nor recuparetation relevant) $\Rightarrow$ $P = 15.1\,\text{kw}$, hence $W = 15.1\,\text{kWh}$ for the one-hour drive. Notice that, because of not perfect battery and motor efficiencies, the energy demand for a BEV with this specification is higher, see 12.5 below

Fuel flow and instantaneous consumption per distance unit

Specific consumption (in [kg/J] if C is measured in kg or [l/J] if C measured in liters):

$$C_{\text{spec}} = \frac{C}{W_{\text{mech}}}$$

Engine efficiency with w_{cal}: energy density (in [J/kg] if C is measured in kg, in [J/l] if C s measured in l)

$$\gamma = \frac{W_{\text{mech}}}{W_{\text{chem}}} = \frac{W_{\text{mech}}}{w_{\text{cal}} C}$$

Relation between C_{spec} and efficiency:

$$C_{\text{spec}} = \frac{1}{\gamma w_{\text{cal}}}$$

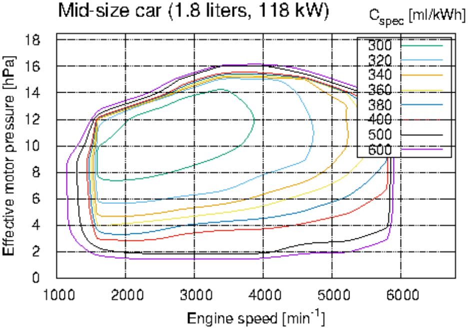
(How to derive this?) insert C_{mech} from the second formula into the first

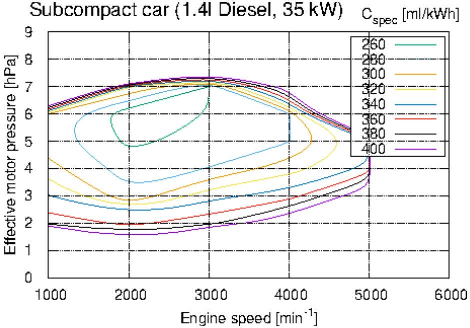
Fuel flow rate:

$$\dot{C} = C_{\text{spec}} P$$

How to derive this? just take the time derivative of the rhs. and lhs. of $C = C_{\text{spec}} W$

Characteristic engine maps





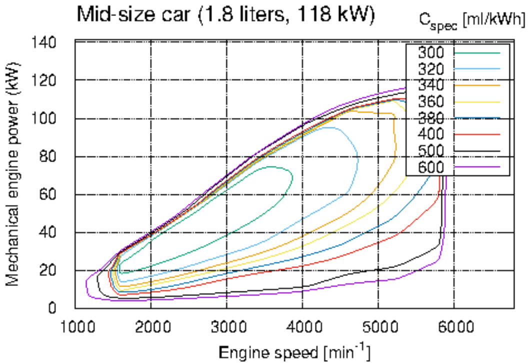
The efficiency of the **internal combustion engine** of ICVs is highly variable and empirically determined as a **characteristic map** on engine test benches

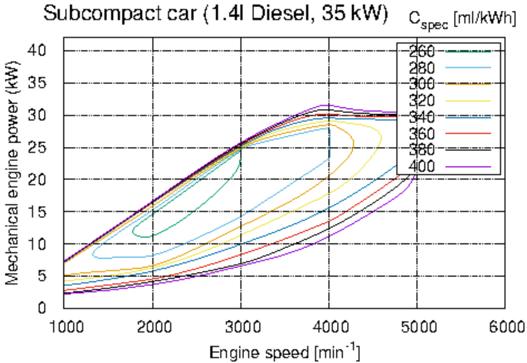
- x axis: engine speed f, often given in rpm (rotations per minute)
- y axis: effective motor pressure $\overline{p}$: proportional to the engine torque M on the crankshaft:

$$M = \frac{\overline{p} V_{\text{zyl}}}{4\pi}$$

  and roughly proportional to how much you push the throttle pedal
- Contour lines: specific consumption in kg/kWh or ml/kWh

Characteristic engine maps with power as independent variable





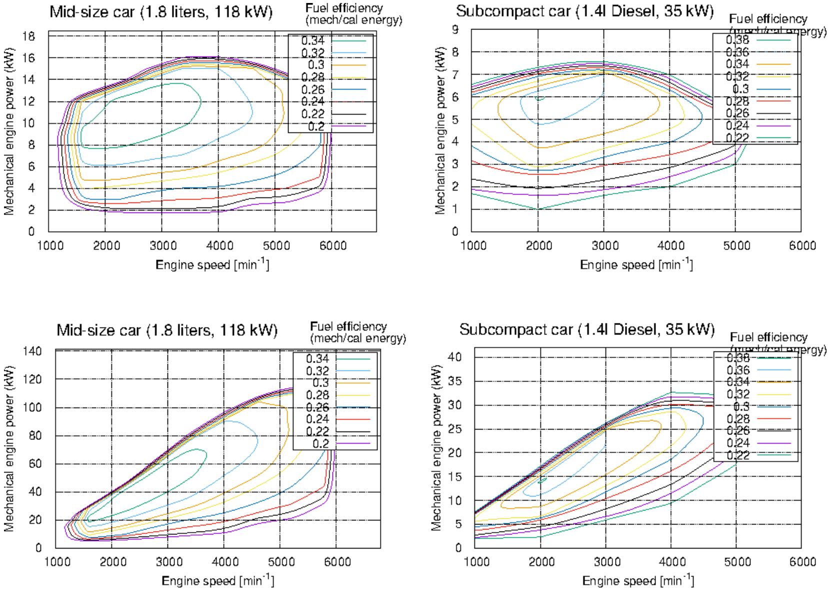
Power for four-stroke engines with cycle time $\tau = 1/f$: $P = \overline{p} V_{\text{zyl}}/(2\tau)$:

$$P(f, \overline{p}) = \frac{\overline{p} V_{\text{zyl}} f}{2}$$

Setting equal $P = M\omega = 2\pi M f$ with the above $\Rightarrow$ already mentioned torque-pressure relation:

$$M = \frac{P}{2\pi f} = \frac{\overline{p} V_{\text{zyl}} f}{4\pi}$$

Efficiency maps

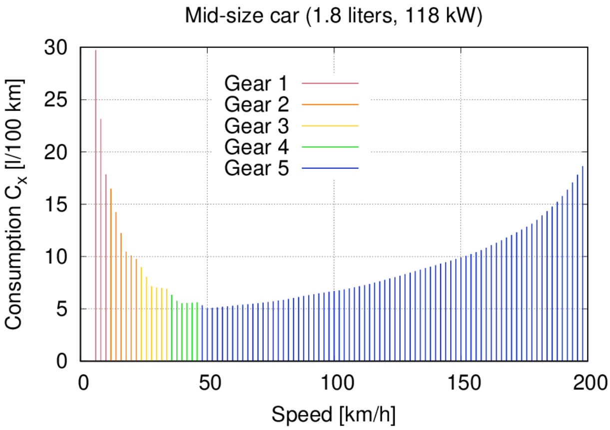


The contour lines for the specific consumption are simply replaced by efficiency contour lines according to

$$\gamma = \frac{1}{C_{\text{spec}} w_{\text{cal}}}$$

(w_{cal} here related to the volume)

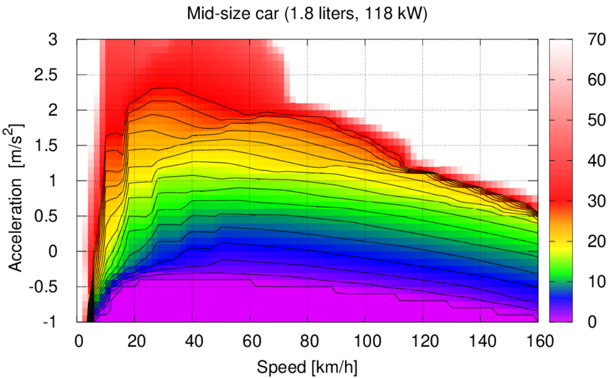
Application 1: Consumption per distance at constant speed



Fuel consumption per $L = 100\,\mathrm{km}$ on a level road:

$$\begin{aligned} C_L &= T\dot{C} = \frac{L}{v}\dot{C} = \frac{L}{v}C_{\text{spec}}P \\ &= LC_{\text{spec}}\left(\frac{P_0}{v} + F\right) \\ &= LC_{\text{spec}}\left(\frac{P_0}{v} + \mu m g + \frac{1}{2}c_d \rho A v^2\right) \end{aligned}$$

General consumption for any vehicle state

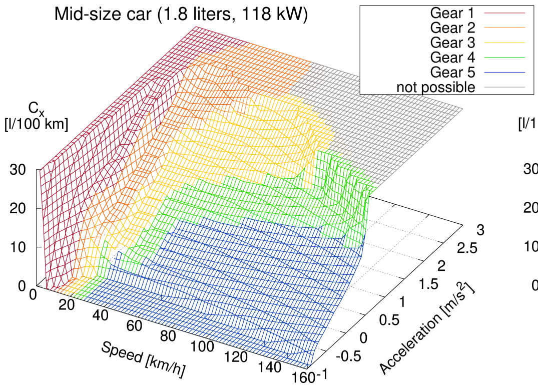


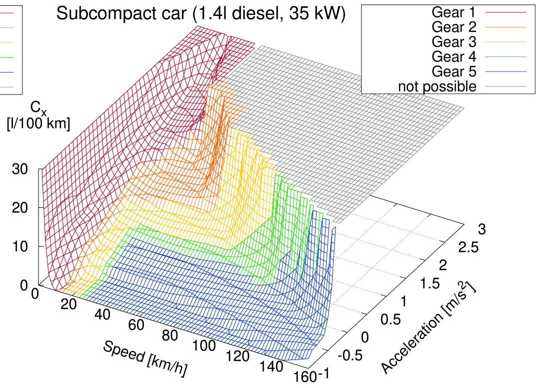
General expression for C_L including accelerations $\dot{v}$, gradients α, and gears g:

$$C_L = LC_{\text{spec}}(f(g,v),P)\frac{P}{v}, \qquad P = P_0 + m_{\text{dyn}}\dot{v}v + mg(\mu+\alpha)v + \frac{1}{2}c_d\rho A v^3$$

The engine speed f is proportional to the vehicle speed v and the gear-specific total transmission ratio i_g between crankshaft and tyre rotation: $f = i_g v/(2\pi r_{\text{tyre}})$ with i_g between about 15 (1st gear) and 3 (highest gear)

General consumption for the fuel-optimal gear





- Generally, several gears are possible for a given vehicle speed and power-demand combination
- In most cases, the highest possible gear is the best one.
- When needing more power at a given speed (uphill gradient, acceleration), a lower gear is often needed, even from a consumption perspective

Questions

? Assuming a constant specific consumption, derive the speed where a vehicle needs the least fuel per 100 km (no gradient, constant speed)

! With constant C_{spec}, the consumption rate is directly proportional to the total power, $\dot{C} \sim P$ and the consumption per distance unit proportional to

$$\frac{P}{v} = \frac{P_0}{v} + \mu m g + \frac{1}{2} c_d A \rho v^2$$

Calculate the argument of the minimum with respect to speed in the usual way

? Why are there gray ranges (not possible) in the maps for the fuel-optimal gear

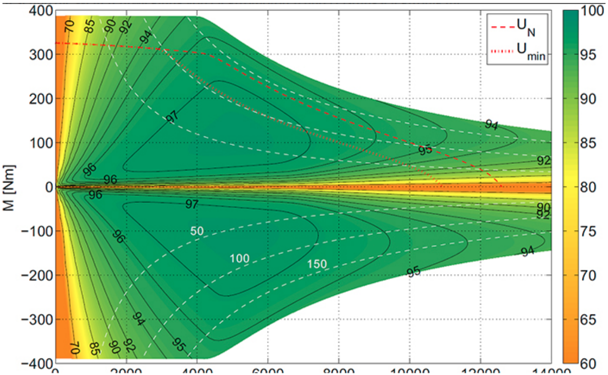
! to much power demand or too high speed ($f > f_{\text{max}}$ even for the highest gear)

? Which rule for fuel-economic driving can be derived from the maps

! The *sweet spot* is at a rather high effective pressure, i.e., throttle pedal pressure and a low engine speed, so choose the highest gear possible which also means pushing more on the pedal compared to a lower gear (see the power maps)

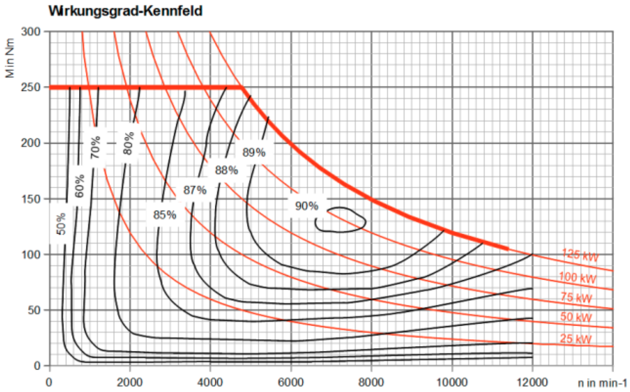
12.5 Electric Vehicles

Battery-electrical vehicles (BEV) can be analyzed by the above physics-based models. The differences are



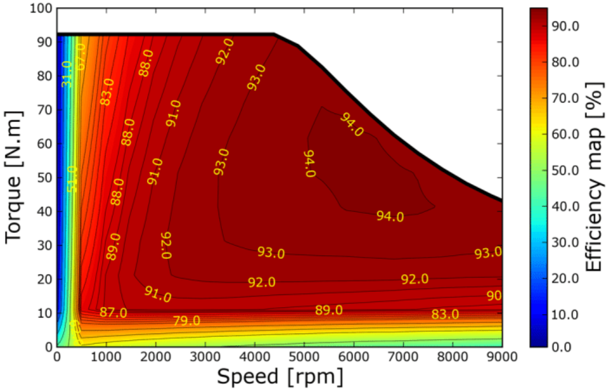
- The engine efficiency is no longer the ratio between mechanical and thermal energy but between mechanical and electrical energy
- The characteristic map is two-sided with a nearly symmetrical generator regime
- The motor efficiency is much higher and nearly constant in a wide range but there is an additional battery efficiency for charging/discharging
- BEV have only a single gear and no clutch (transmission ratio $i_{\text{el}} \approx 9$)

Limits/regimes of an BEV electrical motor



- Low engine speeds: maximum torque limited by the maximum current of the electrical motor
- High engine speeds: maximum power limited by motor overheating
- maximum motor speed: limited by the voltage and mechanical parts
- The first two limits can be exceeded for a short time

Questions on a very small BEV



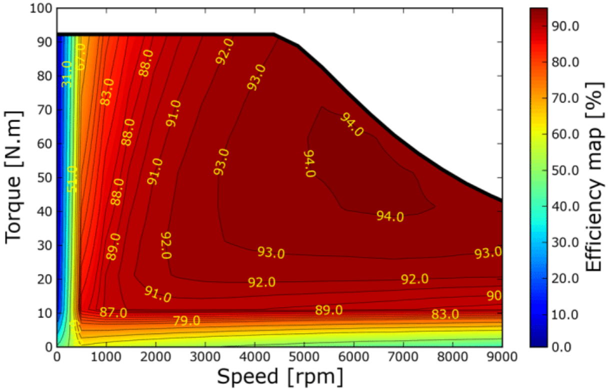
? Give the maximum torque, the maximum engine speed, and the maximum power

! $M_{\text{max}} = 92\,\text{Nm}, \ f_{\text{max}} = 9\,000\,\text{min}^{-1} = 150\,\text{s}^{-1}$, $P_{\text{max}} = 2\pi f_{\text{max}} 43\,\text{Nm} = 40\,\text{kW}$

? The BEV has tyres of an effective outer radius of $r_t = 30\,\text{cm}$ and a transmission ratio $i_{\text{el}} = 9$. Give the cutoff speed where the acceleration starts to drop strongly, and the maximum speed

! We have $v(f) = 2\pi f_t r_t = 2\pi f r_t / i_{\text{el}}$ so, with a cutoff engine speed $f_c = 4\,500/60\,\text{s}^{-1} = 75\,\text{s}^{-1}$ and $f_{\text{max}} = 2 v_c = 150\,\text{s}^{-1}$, we have $v_c = 15.7\,\text{m/s} = 56\,\text{km/h}$ and $v_{\text{max}} = 31.4\,\text{m/s} = 113\,\text{km/h}$.

Questions (ctned)



? The BEV from above has $m \approx m_{\text{dyn}} = 1\,800\,\text{kg}$ and $\mu = 0.015$. The power P_0 is taken directly from the battery. Give the maximum acceleration

! The maximum acceleration is attained near zero speed where air drag is negligible. From the power formula we obtain

$$\dot{v} = P/(mv) - g\mu$$

Since, for $v < v_c$, we also have $P_{\text{max}}(v) = 2\pi M_{\text{max}} f = M_{\text{max}} i_{\text{el}} v / r_t$, speed cancels out, so

$$\dot{v}_{\text{max}} = M_{\text{max}} i_{\text{el}} / (r_t m) - g\mu = 1.7\,\text{m/s}^2$$

The comparatively low value is due to this tiny engine. Typically, because of the high torque M_{max} down to zero engine speed, the BEV accelerations are comparatively high.

Battery charge/discharge rate

BEVs take the basic power demand P_0 directly from the battery instead of from the ICV generator:

$$\dot{W}_{\text{batt}} = \begin{cases} \frac{-P_{\text{el}}}{\eta_{\text{batt}}} & P_{\text{el}} \geq 0 \\ -\eta_{\text{batt}} P_{\text{el}} & P_{\text{el}} < 0 \end{cases}, \qquad P_{\text{el}} = \begin{cases} P_0 + \frac{P_{\text{drive}}}{\eta_{\text{M}}} & P_{\text{drive}} \geq 0 \\ P_0 + \eta_{\text{M}} P_{\text{drive}} & P_{\text{drive}} < 0 \end{cases}$$

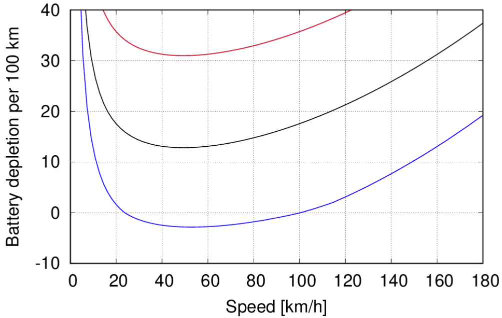
- $\dot{W}_{\text{batt}}$: charge/discharge rate of the battery during driving
- P_{drive}: just the usual product driving force F times speed v
- η_{batt}: efficiency of the battery at charging and discharging
- η_{M}: motor efficiency in both powering and generating mode (characteristic map)
- The roundturn recuperation efficiency is (neglecting the special case $(P_{\text{el}} > 0) \cap (P_{\text{drive}} < 0)$):

$$\eta_{\text{rec}} = \eta_{\text{batt}}^2 \eta_{\text{M}}^2 \approx 0.6$$

Range

Discharge when driving the distance L at constant speed on a level road:

$$\begin{aligned}\Delta W_{\text{batt}}(L) &= -P_{\text{batt}} T \\ &= -P_{\text{batt}} L/v \\ &= -\frac{L}{v}\left(\frac{P_0 + P_{\text{drive}}(v,0,0)/\eta_{\text{M}}}{\eta_{\text{batt}}}\right)\end{aligned}$$



How to calculate the needed kWh per 100 km with this formula? Multiply the above formula for $L = 100\,000\,\text{m}$ with $1/3\,600\,000\,\text{kWh/Ws}$

In spite of recuperation, the discharge per km with changing speeds + stops is higher than when driving at the constand average speed because:

- the nonlinearity $\propto v^3$ of the wind drag increases the average power to overcome it
- recuperation is not perfect
- stops add extra depletion $\Delta W = -T_{\text{stop}} P_0/\eta_{\text{batt}}$